

Snapshottable Stores*

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We say that an imperative data structure is *snapshottable* or *supports snapshots* if we can efficiently capture its current state, and restore a previously captured state to become the current state again. This is useful, for example, to implement backtracking search processes that update the data structure during search.

Inspired by a data structure proposed in 1978 by Baker, we present a *snapshottable store*, a bag of mutable references that supports snapshots. Instead of capturing and restoring an array, we can capture an arbitrary set of references (of any type) and restore all of them at once. This snapshottable store can be used as a building block to support snapshots for arbitrary data structures, by simply replacing all mutable references in the data structure by our store references. We present use-cases of a snapshottable store when implementing type-checkers and automated theorem provers.

Our implementation is designed to provide a very low overhead over normal references, in the common case where the capture/restore operations are infrequent. Read and write in store references are essentially as fast as in plain references in most situations, thanks to a key optimisation we call *record elision*. In comparison, the common approach of replacing references by integer indices into a persistent map incurs a logarithmic overhead on reads and writes, and sophisticated algorithms typically impose much larger constant factors.

The implementation, which is inspired by Baker's and the OCaml implementation of persistent arrays by Conchon and Filliâtre, is both fairly short and very hard to understand: it relies on shared mutable state in subtle ways. We provide a mechanized proof of correctness of its core using the Iris framework for the Coq proof assistant.

CCS Concepts: • **Theory of computation** → **Data structures design and analysis**; **Program verification**; • **Software and its engineering** → *Functional languages*.

Additional Key Words and Phrases: Backtracking, Persistence, Semi-persistence, Program verification.

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1 Introduction

1.1 Snapshots as a Library

Consider an implementation of the Union-Find data structure offering the following interface:

```
type 'a node
val node : 'a -> 'a node
val find : 'a node -> 'a node
```

***Appendices included:** This is a long version of the present paper, with all appendices.

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```

val union : ('a -> 'a -> 'a) -> 'a node -> 'a node -> unit
val equal : 'a node -> 'a node -> bool
val get : 'a node -> 'a

```

A Union-Find graph lets the user incrementally specify an equivalence relation between its nodes, and efficiently query information about the equivalence classes. In our API, each equivalence class carries a value at some type 'a. The user can grow the equivalence relation by unifying two nodes (`union`), providing a merge function for the carried values. Unification is a destructive operation; it modifies the nodes in-place. We can ask for a representative in each equivalence class (`find`), check if two nodes belong to the same class (`equal`), and ask for the value carried by the class (`get`).

A typical implementation would use a data structure such as follows:

```

type 'a node = 'a data ref
type 'a data =
  | Link of 'a node
  | Root of { rank: int; v : 'a }

```

A node is just a mutable reference to some data, which indicates whether it currently is the representative of its equivalence class, or points to another node closer to the representative. The rank integer is used to decide who to elect as the new representative when merging two nodes.

Union-Find is a central data structure in several algorithms. For example, it is at the core of ML type inference, which proceeds by repeated unification between type variables. Union-Find can also be used to track equalities between type constructors, as introduced in the typing environment when type-checking Guarded Algebraic Data Types (GADTs) for example.

When using a Union-Find data structure to implement a type system, it is common to need backtracking, which requires the inference state to be snapshottable. For example:

- (1) A single unification between two types during ML type inference translates into several unifications between type variables, traversing the structure of the two types. If we discover that the two types are in fact incompatible, we fail with a type error. However, we may want to revert the unifications that were already performed, so that the error message shown to the user does not include confusing signs of being halfway through the unification, or so that the interactive toplevel session can continue in a clean environment.
- (2) Production languages unfortunately have to consider backtracking to implement certain less principled typing rules: try A, and if it fails revert to a clean state and try B instead.
- (3) GADT equations are only added to the typing environment in the context of a given match clause, and must then be rolled back before checking the other clauses.

We have encountered requirements (1) and (2) in the implementation of the OCaml type-checker, and (1) and (3) in the development of *Inferno* [Pottier, 2014], a prototype type-inference library implemented in OCaml that aims to be efficient.

Now a question for the reader: how would you change the Union-Find implementation above to support snapshots? The API needs to change a bit to let users talk about the whole Union-Find graph – otherwise, they cannot even ask to go back to a previous version of the graph. The following would be suitable, while still retaining the imperative flavor of the existing API:

```

type graph
type 'a node

val node : graph -> 'a -> 'a node
val get : graph -> 'a node -> 'a
val union : graph -> ('a -> 'a -> 'a) -> 'a node -> 'a node -> unit

```

```
val equal : graph -> 'a node -> 'a node -> bool
```

```
type snapshot
```

```
val capture : graph -> snapshot
```

```
val restore : graph -> snapshot -> unit
```

A first idea to approach our question is to browse the scientific literature for implementations of Union-Find with backtracking, for example looking at [Apostolico, Italiano, Gambosi, and Talamo \[1994\]](#). You would learn that there are algorithms in $O(\log n / \log \log n)$ amortized running times, and then deal with the rewarding but sizeable work of turning a dense 40 pages algorithmic paper from the 90s into runnable code. (This works because Union-Find is a well-studied problem, you would be less lucky with the same question on another, less common mutable data structure.) Unfortunately, we are too lazy to do this. We would like a *generic* approach to add snapshots to an imperative data structure, that does not require expert-level data structure knowledge.

There are two standard generic solutions that can be implemented with relatively little effort.

Full copy : take a snapshot by doing a full copy of the Union-Find graph.

This approach performs well in the case where snapshots are rare – in the extreme case where no snapshots are taken, there is zero overhead. But it can become a performance disaster when snapshots become more frequent, and the number of nodes modified between two snapshots is small – you copy all the nodes, but only touch a few of them. In one of our use-cases using [Inferno](#), this approach makes type-inference 50× slower.

Full persistence : implement the graph on top of a pure, persistent data structure. A standard approach is to change the type 'a data **ref** to become just an int index into a persistent integer map. Implementing capture/restore is then trivial, a snapshot is just the persistent map itself. See for example the Haskell library [disjoint-set](#). However, this adds a logarithmic overhead to each access or modification. In [Inferno](#), we observed that this typically makes type inference about 3× slower, even in cases where no backtracking is used. (Performance is the reason why we stick to an imperative API instead of providing a functional API where modification leaves the input state unchanged and returns an updated state.)

We present a new Store library, which provides generic snapshotability while performing well in all situations. “Snapshots: easy and cheap”. Unlike full persistence, it introduces no overhead when backtracking is absent or infrequent. Unlike full copy, it performs well when backtracking sections touch only a small subset of the structure.

Using our library for Union-Find requires changing the datatype definitions as follows:

```
type 'a node = 'a data Store.Ref.t
```

```
type 'a data =
```

```
  | Link of 'a node
```

```
  | Root of { rank: int; v : 'a }
```

The only change here is to replace the standard 'a **ref** type of OCaml mutable references by the type 'a Store.Ref.t of store references in our Store library, which supports snapshots. In the rest of the code, our Union-Find implementation would need to keep a store in its graph value, and pass this store to the get and set operations on store references. These are trivial changes.

Summary. Our Store library introduces a notion of *store*, a bag of mutable references that lets you capture and restore the state of all its references at once. Store can be used to easily make arbitrary mutable data structures snapshottable, by replacing their mutable pointers by store references.

1.2 Notions of Persistence

The standard notion of *persistence* used in the algorithmics literature is one where modification operations return a different version of the data structure, without modifying the version provided in input. There are in fact many nuances to persistence, described below.

functional data structures are fully immutable, as is idiomatic in functional programming languages. (Demaine, Langerman, and Price [2008] call them *functional*, one may also call them *pure* data structures.) They typically rely on sharing immutable substructures between different versions, and *copying* the paths from those shared substructures to the root of the structure.

Functional data structures have the advantage that they are thread-safe by construction: they can be accessed in parallel without any synchronization.

persistent data structures may be implemented using mutable state; a typical example would be the Splay-tree data structure that performs imperative rebalancing under the hood. They may not be thread-safe. In the case of our store, our persistent snapshots are persistent in this sense, and in particular they are not thread-safe – we cannot support restoring two snapshots in parallel.

partial persistence is a weaker notion of persistence where only the “last” version of the data structure may be updated, but read-only queries may be performed on arbitrary versions of the structure. We could expose this capability for our backtracking stores, but we do not have a clear use-case that would justify the additional implementation complexity.

confluent persistence is a stronger notion of persistence where two independent instances of a persistent data structure may be merged together – for example, merging two persistent sets or maps together. Some persistent data structures cannot offer confluence at a reasonable cost. We have not implemented confluence for our stores; the user has to plan in advance and allocate the separate data structures in the same underlying store.

semi-persistence is a weaker notion of persistence where only a linear chain of versions is maintained at any point in time, rather than a tree of versions in the general case: acting on a past version invalidates all the versions that are “after” this past version, and we cannot access them anymore.

Our store provides persistent snapshots and also exposes a semi-persistent API based on *transactions* that we describe in Section 4. This brings moderate performance benefits for use-cases that do not need full persistence; we observed no improvement on some benchmarks, 5%-10% speedups in others, and larger gains for some very specific workloads.

Use cases for persistence and semi-persistence. A semi-persistent approach suffices whenever we only ever restore ancestors of the current version. This is the case for most backtracking problems. For example, in a SAT/SMT solver, backtracking (when a conflict is found) goes back to a time when fewer decisions were made, it never jumps “forward” into a saved search state where more decisions had been made.

Some search algorithms do not perform a full depth-first search, they explore several positions in the tree in parallel, iteratively refining the more promising positions, and they may “fork” new search branches from the same promising position several times. Those require persistent snapshots. Another trite example is *saves* in video games, where players can load previous saves to move forward in game time, or go back to parallel/divergent play histories.

The original persistence use-case of Baker [1978] was the implementation of efficient dynamic binding in a Lisp interpreter. Efficient Lisp interpreters at the time would have a semi-persistent store for the dynamic environment, with a stack structure mirroring the dynamic call stack of the program – on function return they would “undo” bindings performed within the body of the

function, to return to the dynamic binding environment of the caller. But this approach does not work when returning functions as first-class values, as the body of the functions (when called later) should be evaluated in the dynamic environment where it was defined, whose definitions have been undone in the meantime. Instead, Baker implemented a persistent store for its environments; first-class functions would capture a snapshot at their definition site, to be restored at call-time.

1.3 Performance Model

Following Baker [1978], we implement Store as a “journalled” data structure; the current version of the store is represented in memory just like normal references, but we also keep a record of past operations to be able to go back to previous versions. If the log of operations between two snapshots A and B has size Δ , then the space cost of the log is $O(\Delta)$, and restoring the state of A when we are currently at B takes times $O(\Delta)$.

One may expect the number Δ of operations recorded to be exactly the number of operations performed between the two snapshots. For Union-Find problems the number of reference updates remains relatively small, but in general this number of operations may be large, much larger than the size of the data structure itself. We introduce a key optimization, *record elision*, where we record at most one operation per store location updated between two consecutive snapshots. As a result, our bound Δ is the number of distinct locations modified between the two snapshots, which could be much smaller than the total number of operations. Record elision does not just improve asymptotics, it is key to low-overhead implementation of set for store references.

We can benefit from record elision because our interface requires users to be explicit about where they take snapshots, that is, where the backtracking points are in their programs. Record elision is not available to the more elegant, more convenient and more functional interface of a persistent store, which corresponds to taking a snapshot after each update operation.

In the specific case where each snapshot is restored at most once – this is a common property of backtracking workloads, and enforced by our *semi-persistent* interface – one can amortize the cost of snapshot restoration over each operation after the snapshot is taken, so restoring a snapshot has $O(1)$ amortized complexity. This amortization does not work in the general case of persistent snapshots; for example, one could keep alternating between two snapshots without performing any operation in between. This bad interaction between persistence and amortized bounds is a well-known problem in the algorithms literature, typically solved by sophisticated *rebuilding* techniques [Chuang, 1994, 1992]. We do not solve it, as our current use-cases do not need it.

When discussing our design choices, we mention constant factors a lot. Imagine that you are implementing a type checker (with type inference) for your programming language, and suddenly you realize that an oddball new feature F that you want requires backtracking inference decisions, which you did not need previously. You have to move your type-checker state to different data structures that support snapshots. You need this new capability only for programs that use feature F , but you pay the cost of the data structure all the time.¹ If you are not careful about constant factors, this implementation change could make your type-checker $2\times$, $5\times$ or $\log(n)\times$ slower for *all* programs, whether they use your new feature or not. This is not acceptable.

Contributions

We report on the implementation of *snapshottable stores*, a bag of mutable references that support efficiently capturing and restoring its state to implement backtracking. This abstraction can be used to easily add snapshots to complex imperative data structures. The implementation (1) is expressive,

¹You could think of dynamically switching from one data structure to another when feature F occurs. This increases implementation complexity, and you still have the problem of not-too-slow type inference for programs that do use F .

it provides persistent and not just semi-persistent snapshots, (2) is efficient, as demonstrated by benchmarks, and (3) its core mechanism is formally proved correct.

We claim the following contributions:

- (1) The concept of “snapshottability” as a service worth providing in a reusable, generic way as a small software library. When we looked at existing library ecosystems (in OCaml but also Haskell, Scala, etc.) we found a few implementations of snapshottable stores in the wild, but almost always as part of a larger program that uses it exclusively, not as a shared library.
- (2) An efficient OCaml implementation of a store with persistent snapshots [Clément and Scherer, 2023]. The implementation, extending the journaled approach of Baker [1978], is short and subtle. It is *heterogeneous*, references of different types can be tracked by the same store.
- (3) The *record elision* optimization which is key to an almost-zero overhead on the set operation on set-heavy workloads. Forms of record elision exist in previous semi-persistent implementations, but combining persistent snapshots and record elision is challenging and Store is the first implementation to do so.
- (4) A mechanized proof of correctness of persistent snapshots in the presence of record elision, using the Iris separation logic framework in the Coq proof assistant [Allain, 2024].
- (5) An additional API of *semi*-persistent snapshots, which restricts ourselves to a linear history of snapshots for further efficiency benefits.
- (6) Benchmarks comparing the performance of our implementation with other approaches, demonstrating that Store performs well on a broad variety of workloads.

2 A Core Store

2.1 Baker’s Version Trees

The starting point of our implementation is Baker’s *version trees* introduced in Baker [1978]. Baker’s trick has been reused or rediscovered many times since, mostly in the context of implementing persistent *arrays*: homogeneous structures indexed by small integers. O’Neill and Burton [1997] give a pleasant survey of approaches to persistent arrays. It lists three works that reinvented Baker’s trick in the late 80s.

In Baker’s work, the programmer can refer to many different persistent versions of a data structure, but one is the “current version” on which access and update operations operate as usual in constant time. The “current version” uses its standard representation – for example, the current version of a Baker array is just an array. Older versions are represented by nodes in a version graph (in fact a rooted tree), whose root is the current version, and where edges log operations that were performed. Any older version can be restored by applying a “rerooting” operation on its node (it becomes the new root of the graph) which reverts all the updates that happened between that older version and the current version.

Consider the following Store user program:

```
let s = Store.create () in
let r = Store.Ref.make s 0 in
let snap0 = Store.snapshot s in
let () = Store.Ref.set s r 1 in
let () = Store.restore s snap0 in
let () = Store.Ref.set s r 2
```

At the point of `let r = Store.Ref.make s 0`, our version tree (shown in Figure 1a) has a single node where the reference `r` has value `0`. The mapping $\{r \mapsto 0\}$ is not stored within the node

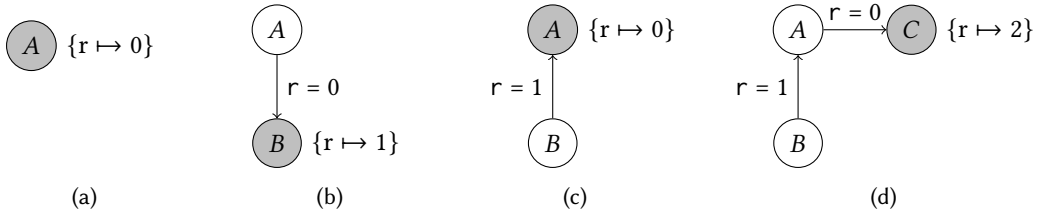


Fig. 1. Version trees in the example program

A , it describes the current state of the reference r in the current state. We place it on A to indicate that A is the current root of the version tree, which is also indicated by the darker background.

Calling `Store.snapshot s` at this point does not change the version tree. The snapshot returned is basically just a pointer to the current root of the graph, B .

Calling `Store.Ref.set s r 1` creates a second node B in the version tree, which describes the new current state (see Figure 1b). The node A now points to B , with information on how to revert to A if desired – one should restore r to \emptyset .

Calling `Store.restore s snap0` *reroots* the version tree to have root A again – A was the current node at the time where `snap0` was captured (see Figure 1c). We do this by starting from the snapshot node A , updating the current state by using the information stored on the edges. Note that the edge between A and B has changed direction (now B points to the new current root A), and the information on the edge now describes how to restore the state of B from the state of A .

At this point, calling `Store.set s r 2` creates a new node C from A , which becomes the new current root, as shown in Figure 1d.

This representation provides constant-time access to the current state of the store, with the exact same constant factors as OCaml native references – r can in fact just be a native reference.

A snapshot is just a node in the version tree. Restoring the snapshot means rerooting the tree so that the snapshot node becomes the new current root – and the current state gets updated accordingly. We sketch our implementation in Section 2.3. It is linear in the length of the path from the snapshot node to the current root node. The length of this path is the number of operations that happened “after” the snapshot node, in a sense that is made precise in the next section.

2.2 A Whiff of Graph Theory

In graph theory, an (undirected) tree is a certain kind of (undirected) graph: a graph that is acyclic (no cycle in the graph) and connected (all nodes are reachable from each other). In other words, an *undirected tree* is an undirected graph where there exists a unique path between all pairs of nodes.

The notion of “tree” that is common in programming corresponds to the notion of *rooted tree* in graph theory, a tree with a designated root node. The choice of root uniquely determines a *parent relation* that relates A to B when the parent of A is B . There is at most one parent, and the root is the only node with no parents. If we look at a given undirected tree T , and two different choices of root M and N , there is a simple relation between the parent relations of the M -rooted and N -rooted trees: all nodes have the same parent in both trees, except on the (unique) path from M to N where the parent relations are mutual inverses.

Over our version trees, there are two rooted trees (two choices of root) of interest:

- (1) The *current tree*, whose root corresponds to the current state of the structure – C at the end of our example above.

- (2) The *historic tree*, whose root is the initial node created when the store was created – A in our example. (This is a slight simplification, there is a version tree node before r was created that we are not showing in the version tree for simplicity.)

We call *history* of a node the path from this node to the historic root. The complexity of rerooting from the current tree A to a given snapshot tree B is exactly the length of the unique path from A to B in the version tree.

2.3 Implementing Version Trees

We learned of Baker’s trick from [Conchon and Filliâtre \[2007\]](#), which use it to define persistent arrays, on top of which they build a persistent Union-Find, with OCaml code fairly close to what we show in this section. The core of Store, described here, has the following API:

```

type store
val create : unit -> store

module Ref : sig
  type 'a t
  val make : store -> 'a -> 'a t
  val get : store -> 'a t -> 'a
  val set : store -> 'a t -> 'a -> unit
end

type snapshot
val capture : store -> snapshot
val restore : store -> snapshot -> unit

```

The Ref module implements mutable references inside the store. The store must be passed as argument to all operations on references, and it is an *unchecked* programming error to use a reference with a store it does not belong to. The snapshot type represents persistent snapshots of the state of the store at a given point in time. New snapshots for the current state are created with capture, and the store state can be later reset to the snapshot state using restore.

The version tree is a graph of mutable nodes, whose value can be Mem to indicate that they are the current root – the state of this node is stored in memory – or Diff if they log a reference write.

```

type node = data ref    and data = Mem | Diff : 'a Ref.t * 'a * node -> data

```

If A has B as parent in the current tree, its data must be $\text{Diff}(r, v, B)$, where r is a reference and v is the value of r , in A .

Finally, the store is just a mutable reference to the current root of the version tree, and a snapshot remembers which node was the current root when it was captured².

```

type store = { mutable root : node; }    type snapshot = { root : node; }

```

Easy parts. Creating a new store or taking a snapshot are the obvious things:

```

let create = { root = ref Mem }
let capture store : snapshot = { root = store.root }

```

²In the actual implementation, we also remember the store, in order to fail at runtime if the user tries to use a snapshot with another store.

References have the same representation and get operation as standard OCaml references:

```

module Ref = struct
  type 'a t = { mutable value : 'a; }
  let make v = { value = v }
  let get _s r = r.value
  let set s r v = ... (* to be detailed below *)
end

```

The two difficult operations are `Ref.set`, which grows the version tree with a new node, and `restore`, which reroots the version tree to a snapshot node.

Update operation: `Ref.set`. When we call `set s r v`, the current root of the version tree, which was previously a `Mem` node, becomes a `Diff` node pointing to a *new* current root. The `Diff` node carries the *previous* value of the reference, to be able to restore the reference to its previous value later on.

```

let set s r new_val =
  let old_val = r.value in
  let new_root = ref Mem in
  let old_root = s.root in
  r.value <- new_val;
  old_root := Diff(r, old_val, new_root);
  s.root <- new_root

```

The code is short, but reasoning about it is difficult. It helps to define a *model* of the current store, and a model of each node in the version tree. A node A is modelled by a functional *mapping*, denoted $\llbracket A \rrbracket$, from references to their values. The model of the store is the *current model*, the model of the current root. The model of each node is defined as follows:

- (1) The mapping of the `Mem` node maps each store reference to its current value.
- (2) The mapping of a `Diff(r, v, n)` node is $\llbracket n \rrbracket[r \mapsto v]$.

In other words, if B is the parent of A in the current tree, then the edge from A to B (stored in A 's data in the OCaml representation) records how to transform $\llbracket B \rrbracket$ into $\llbracket A \rrbracket$.

If we look at `Ref.set` again, we can now check that, given a current mapping m , `set s r v` will move us to a new current mapping $m[r \mapsto v]$ (with `r.value <- new_val`). Furthermore, since `old_val` stores the value $m(r)$, the mapping of the old root (and hence of the existing version tree) is preserved as it becomes $m[r \mapsto \text{new_val}][r \mapsto \text{old_val}] = m[r \mapsto m(r)] = m$.

Reroot, restore. The operation `reroot(A)` makes an arbitrary node A the new root of the current tree – without changing the model of any snapshot node in the tree. A “simple” implementation of `reroot` is shown in Figure 2.

Our actual (verified) implementation contains two improvements over this “simple” version.

- (1) In this version, every recursive call in the `Diff(r, v, n')` case writes the data of both the node n and of its parent node n' – which becomes its child in the modified version tree. This means that the data of most nodes is written twice, first to `Mem` and then to their final data. Our implementation avoids these redundant modifications by writing `Mem` only once at the end, at the cost of a more complex specification for the recursive function.
- (2) `reroot` reverts and reverses `Diff` nodes from the root of the version tree to the snapshot node. This corresponds to undoing operations from the most recent operation to the oldest operation, as it should be. The simple version does this via a non-tail-recursive call `reroot n'` on the

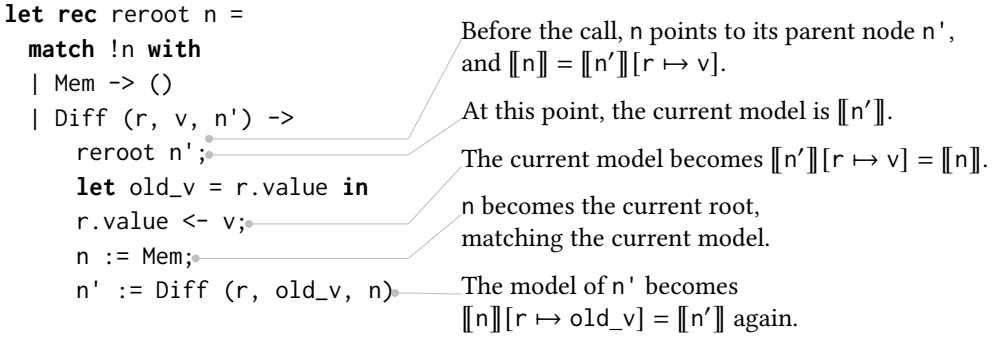


Fig. 2. Reroot (simple version)

parent node n' before it handles the child n . To avoid stack overflows our implementation uses a tail-recursive variant where we first accumulate `Diff` nodes in a list, most recent operation at the head, and then traverse the list in order.

Finally, `restore` can be easily defined from `reroot`:

```

let restore (store : store) snapshot =
  reroot snapshot.root;
  store.root <- snapshot.root

```

Remark. This concludes our retelling of the core algorithm of Baker [1978], with an OCaml realization inspired by Conchon and Filliâtre [2007]. We consider what follows as original work.

2.4 Record Elision

Record elision is a key optimization that changes the qualitative performance profile of the library. The idea is simple: if we have already performed a set operation on some reference r in “the current version” (since the last snapshot), we have created a `Diff` node with the value before that operation; so if we perform a set on that reference again, there is no need to log anything, as the older `Diff` node will already reset the reference to its previous value. This optimization is only valid if no snapshot was taken after the previous `Diff` node, otherwise that snapshot would get the wrong value of r on rerooting.

We do not wish to search the history on each set to check this property. In fact we cannot check it with the previous definitions, as there is no trace in our graph data structure of which nodes have been captured as snapshots. We solve both issues by introducing a notion of *generation*, an integer that counts the number of snapshots taken in the history of a node. In particular, if two nodes belong to the same history and have the same generation, there is no snapshot between them.

We keep track of generations in the store graph (the generation of the current root), in snapshots (the generation of the snapshot node), in references (the generation of the last `Diff` node on this reference), and `Diff` nodes.

```

type store = { mutable root : node; mutable generation : int; }
type 'a Ref.t = { mutable value : 'a; mutable generation : int; }
type snapshot = { store : store; root : node; generation : int; }
type node = data ref
and data = Mem | Diff : 'a Ref.t * 'a * int * node -> data

```

Creating a new snapshot increments the generation of the store:

```
let capture s =
  let snap = { store = s; root = s.root; generation = s.generation; } in
  s.generation <- s.generation + 1;
  snap
```

All the magic happens in the `Ref.set` function which updates a store reference. (We use a lighter gray color for code that is identical to the previous version.)

```
let set (s : store) (r : 'a Ref.t) (new_val : 'a) : unit =
  if s.generation = r.generation
  then r.value <- new_val
  else
    let old_val = r.value in
    let old_gen = r.generation in
    let new_root = ref Mem in
    let old_root = s.root in
    r.value <- new_val;
    r.generation <- s.generation;
    old_root := Diff(r, old_val, old_gen, new_root);
    s.root <- new_root
```

By comparing the two integers `s.generation` and `r.generation`, we check whether a snapshot was captured between the last recorded write to the reference and the current root. If no snapshot was taken, then we do *not* record the new update in the version tree – it is useless, as any restore call will restore an older value of the reference from the recorded write. We call this a *record elision*. If a snapshot was taken, we update the generation of the reference: we have just recorded the write, so we can elide all records for that reference until the next snapshot is taken.

In terms of model, calls to `set r v` where record elision takes place are harder to reason about, because they mutate the mapping of existing nodes in the version tree: for all the nodes from the current root (included) to the last `Diff` node on this reference excluded, their mapping is from some m to $m[r \mapsto v]$. In the absence of record elision, the mapping of all version tree nodes was persistent: the data on the node may change but its mapping remained unchanged. Record elision relaxes this property: the mapping of nodes that are *captured by a snapshot* is persistent, but other nodes, in fact the nodes between the last snapshot and the current root, may see their mapping changed by later operations. This weaker guarantee suffices, as we only provide persistent *snapshots* to users, they cannot observe the mapping change for other nodes.

Performance impact. Record elision has a transformative performance impact on workflows that use `Ref.set` heavily and snapshot capture rarely. (We generally assume that backtracking is rare relative to reads and writes, but many workflows are rather dominated by reads so record elision matters less.) Indeed, a record-elided `Ref.set` is just an integer comparison and a write, which is basically the same as a write: in OCaml, polymorphic writes go through a write barrier, so the cost of the write dominates the generation test. In the regime where most writes are elided, `Ref.set` is essentially as fast as OCaml primitive references, providing the almost-zero overhead we advertised. On the other hand, a non-elided `set` performs an extra write and an allocation. On a `get/set` microbenchmark with 16 `get` for each `set`, disabling record elision made the test 6× slower.

Record elision also has a transformative effect on the asymptotic complexity of store operations. As we detailed in the introduction (Section 1.3), the key complexity parameter of `Store` is the size Δ

of the log between two consecutive snapshots. Without record elision, Δ is the number of write operations that happened since the previous snapshot, which can grow arbitrarily large. Record elision reduces Δ to the number of distinct memory locations touched since the previous snapshot.

Notes. If one tries to implement persistent data structures on top of Store by capturing a snapshot after each write operation, then record elision never applies. This explains why we are not offering a persistent API for Store. It also probably explains why we have not found a description of this simple idea in the existing literature on more-or-less-persistent data structures.

It is tempting to think of generations as unique timestamps for snapshots, and indeed the two concepts overlap in semi-persistent implementations. Scaling record elision to the persistent setting required a more precise definition of generations that need *not* be unique. Preserving uniqueness in the persistence setting would be an instance of the *order maintenance problem*, which has amortized constant-time solutions (Bender, Cole, Demaine, Farach-Colton, and Zito [2002]; but think of the constant factors!) and is a common ingredient in persistent data structure design.

2.5 Liveness

An important consideration in our choice of data structure design is *liveness*. In garbage-collected languages, the memory footprint of a data structure is determined by what other portions of memory it references, keeps *alive*. Suppose for example that a user captures a snapshot of the store, and then later drops all references to this snapshot. Can the memory corresponding to this snapshot be collected, or is it kept alive by the global Store data structure?

The version tree structure inherited from Baker [1978] has excellent liveness properties: pointers in the data representation coincide with the parent relation of the current tree, so that referencing the store only keeps the current root alive. In particular, if we do not reference any snapshot, then the whole version tree (except for the root) can be collected. Locally, only the operations that are needed to restore a snapshot that is still referenced are kept alive. This still holds if the user forgets a reference: as long as a snapshot mentioning it is kept alive, the reference will be kept alive (one could use weak pointers and ephemerons [Hayes, 1997] to get better liveness properties there, at significant complexity and runtime cost). On the other hand, if the user forgets all the snapshots mentioning a reference, then it can be collected. This is a common situation in realistic workloads such as type-checking: we typically forget all the references and all the snapshots created when typing a given subterm.

Another case where our implementation can “leak” values is when forgetting intermediate snapshots: if there are three consecutive snapshots A , B and C with the same reference r being written both between A and B and between B and C , forgetting B will still keep the value of r in B alive even though we can never restore B again. We could consider an implementation using weak pointers and finalizers to notice this and compress the log, but suspect that the cost in performance and code complexity would not be worth it for most applications. Our semi-persistent interface (see Section 4) provides a `commit` operation that does remove some (but not all) such unneeded records.

Most other implementation choices have worse liveness properties. Semi-persistent implementations based on a centralized journal often cannot forget any snapshot. Implementations based on functional or imperative maps (with copy) can never forget references. Another common implementation choice for persistent structures, the so-called *fat nodes* approach, keeps a list of all past values in the reference itself. This makes it impossible to forget past versions or siblings, but it allows the user to forget references.

We considered liveness properties seriously in our design, and it helped guide some implementation choices. We believe that the liveness properties of our implementation are adequate, and that

<p>CREATE $\{\text{True}\} \text{create } () \{ \lambda s. \text{store } s \emptyset \}$</p>	<p>REF $\{\text{store } s \sigma\} \text{ref } s v \{ \lambda r. \ulcorner r \notin \text{dom}(\sigma) \urcorner * \text{store } s ([r := v] \sigma) \}$</p>
<p>GET $\frac{r \in \text{dom}(\sigma) \quad \sigma(r) = v}{\{\text{store } s \sigma\} \text{get } s r \{ \lambda v'. \ulcorner v' = v \urcorner * \text{store } s \sigma \}}$</p>	<p>CAPTURE $\{\text{store } s \sigma\} \text{capture } s \{ \lambda t. \text{store } s \sigma * \text{snapshot } s t \sigma \}$</p>
<p>SET $\frac{r \in \text{dom}(\sigma)}{\{\text{store } s \sigma\} \text{set } s r v \{ \lambda (). \text{store } s ([r := v] \sigma) \}}$</p>	<p>RESTORE $\{\text{store } s \sigma * \text{snapshot } s t \sigma'\} \text{restore } s t \{ \lambda (). \text{store } s \sigma' \}$</p>

Fig. 3. Interface of our Coq store

it does make a positive difference in practice with respect to implementation approaches that keep all store operations alive – in the type-checking use-case, for example.

3 A Coq Store

In this section, we use separation logic to specify and verify the core of our approach: an implementation of snapshottable stores with record elision but without transactions. After introducing the formal setting (Section 3.1), we present the specification (Section 3.2) and the high-level ideas of the proof (Section 3.3). Our results are entirely mechanized in the Coq proof assistant [Allain, 2024].

3.1 Formal Setting

Formally, we use the Iris separation logic framework [Jung, Krebbers, Jourdan, Bizjak, Birkedal, and Dreyer, 2018]. We write our programs in an untyped call-by-value λ -calculus with mutable state, similar to the HeapLang language that comes with Iris.

In the following, we write P for an Iris assertion, $P_1 * P_2$ for separating conjunction, $P_1 \multimap P_2$ for separating implication and $\ulcorner \phi \urcorner$ for the embedding of a pure (meta-level) proposition ϕ . To specify an expression e , we use a Hoare triple $\{P\} e \{ \lambda v. Q \}$, where P is the precondition, meta-variable v captures the resulting value and Q is the postcondition.

3.2 Specification

Figure 3 presents the specification of our Coq store. To describe a store s at the logical level, we use the assertion $\text{store } s \sigma$ denoting that s is modeled by the (partial) mapping σ from references to values. We write $\sigma(r)$ for the value associated to reference r in σ , $[r := v] \sigma$ the functional update of σ with the mapping $r \mapsto v$, and $\text{dom}(\sigma)$ the domain of σ (the set of created references). We first present the specification of the functions `create`, `ref`, `get` and `set`. We then turn our attention to the functions involving snapshots, namely `capture` and `restore`.

CREATE asserts that `create ()` has trivial precondition and returns a store s with an empty model. **REF** asserts that `ref s v` creates a new reference. The precondition consumes an assertion $\text{store } s \sigma$ and the postcondition produces an assertion $\text{store } s ([r := v] \sigma)$, where r is the returned reference. The postcondition also asserts that r is fresh. **GET** asserts that `get s r` returns the value associated to r in the model of s . The precondition consumes an assertion $\text{store } s \sigma$, and requires that r is in the domain of σ and is mapped to the value v . The postcondition asserts that the function returns the value v , and restores the assertion $\text{store } s \sigma$. **SET** asserts that `set s r v` correctly sets the value associated to r to v in the model of r . The precondition consumes an assertion $\text{store } s \sigma$ and requires that r is in the domain of σ . The postcondition produces an assertion $\text{store } s ([r := v] \sigma)$.

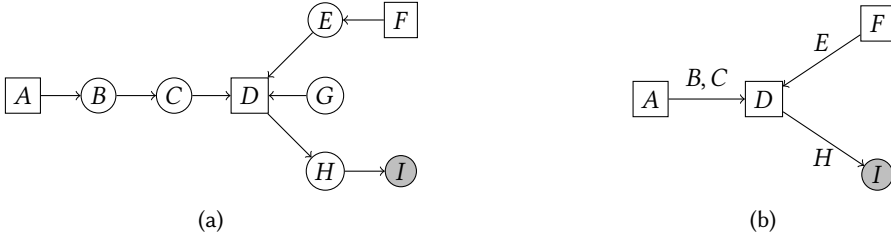


Fig. 4. Graph of nodes and its corresponding subgraph of captured nodes. Squares represent captured nodes and circles non-captured nodes. Gray node identifies the root.

To describe a snapshot t at the logical level, we introduce the assertion snapshot $s t \sigma$. It asserts that t is a valid snapshot of the store s , whose model was σ when the capture occurred. Crucially, the assertion snapshot $s t \sigma$ is *persistent* [Jung, Krebbers, Jourdan, Bizjak, Birkedal, and Dreyer, 2018, §2.3]. A persistent assertion is in particular duplicable, meaning that the following entailment holds: snapshot $s t \sigma * (\text{snapshot } s t \sigma * \text{snapshot } s t \sigma)$.

CAPTURE asserts that capture s creates a new snapshot. The precondition requires that s is a valid store of model σ . The postcondition asserts that the store was preserved and that the function returned a snapshot t such that snapshot $s t \sigma$ holds. **RESTORE** shows that indeed, restore $s t$ updates the model of s to the model captured by t . The precondition consumes the assertion store $s \sigma$ and snapshot $s t \sigma'$, and the postcondition produces the updated assertion store $s \sigma'$. Notice that there is no need to repeat the assertion snapshot $s t \sigma'$ in the postcondition. Thanks to persistence, the user can duplicate the assertion before applying **RESTORE**.

3.3 High-Level Ideas of the Proof

We wrote two different proofs of correctness of Store. In the first iteration, we used the graph structure discussed so far and present in-memory in our implementation, with a mapping σ for each node of the graph. We were able to prove the specification, but without generations and record elision. As we explained in Section 2.4, record elision mutates the mapping of the nodes between the last captured node and the current root, and our attempts to formalize this ran into a wall.

The second iteration of the proof, which supports generations and record elision, relies on a more structured presentation of the graph: the subgraph of captured nodes. More precisely, captured nodes induce a coherent subgraph in which two captured nodes are connected by a chain of non-captured nodes. Consider, for example, the graph of Figure 4a. In the corresponding subgraph shown in Figure 4b, we only retain captured nodes A, D, F . We track separately the root I and the chain leading to it from the last captured node. Even in the presence of record elision, writes only affect the chain to the root, the mappings of the captured nodes remain persistent.

4 Semi-Persistence Through Transactions

4.1 Introduction

The capture and restore API presented in Section 2.3 is low-level in the sense that users have to create persistent snapshots, keep track of them, and restore them manually. For some common workloads, we provide high-level wrappers that are more convenient but also less expressive.

```
val temporarily : store -> (unit -> 'a) -> 'a
val tentatively : store -> (unit -> 'a) -> 'a
```

These wrappers call the provided function, then restore the state of the Store to the state it had prior to the call either unconditionally (temporarily) or if an exception is raised (tentatively).

Both functions can be implemented by capturing a snapshot before calling f , and restoring it after the call if necessary. Snapshots created by these wrappers have interesting properties: not only are they restored at most once, but their use follows a rigid structure dictated by scoping rules. This corresponds exactly to the notion of *semi-persistence* in the data-structure literature: there is a *stack* of versions, and versions that are removed from the stack are no longer accessible. Imposing such a linear (or affine) discipline on snapshots makes reasoning about the implementation easier, and avoids the aliasing of mutable state that makes the implementation of `restore` so subtle (Section 2.3).

One could provide an entirely different implementation of Store that only provides a semi-persistent API. It can be expected to be slightly faster, perhaps simpler to implement, but would provide less functionality than the persistent API of Store. Instead, we describe in this section an extension of the Store API with semi-persistence in the *same* implementation, providing a combination of both capabilities. We call this API *transactional*, because each semi-persistent snapshot (or transaction) is terminated by either keeping (commit) or discarding (rollback) the changes within. Users are expected to stick to the simple persistent API and the convenience wrappers temporarily and tentatively, which are implemented using the semi-persistent API for performance. In more advanced scenarios, users can directly use the transactional API, which is more difficult to use but can bring additional performance improvements.

4.2 Transactions for Semi-Persistence

Besides the high-level wrappers mentioned earlier, the transactional API is as follows:

```
type transaction
val transaction : store -> transaction
val rollback : store -> transaction -> unit
val commit : store -> transaction -> unit
```

A transaction represents an interval in the program execution during which an ephemeral copy of the store is preserved. The transaction is created by calling `transaction`, and terminated by calling either `rollback` or `commit`. `rollback` is similar to `restore` in the persistent API: it resets the state of the store to the one it had when the transaction started. `commit` terminates `transaction`, but the state of the store is unchanged – it merely discards the ephemeral snapshot.

Transactions can be nested following a stack-like discipline. Transactions are *valid* when created, and terminating a transaction invalidates it and all the transactions that were created while it was valid. Using an invalid transaction is a programming error and raises an `Invalid_argument` exception.

As a simple example of use of transactions, we can implement the tentatively convenience wrapper using the transactional API:

```
let tentatively store f =
  let trans = Store.transaction store in
  match f () with
  | v -> Store.commit store trans; v
  | exception exn -> Store.rollback store trans; raise exn
```

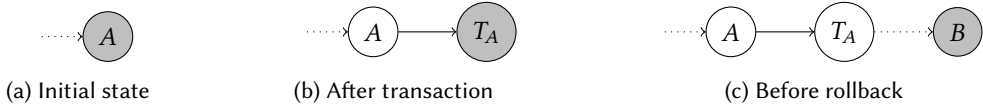


Fig. 5. Version graph during a transaction

4.3 Combining the Persistent and Semi-Persistent APIs

It is possible to write and reason about programs that combine both APIs by viewing the transaction stack as a part of the capturable state of the store. Using a transaction while it is not part of the current transaction stack is a programming error, but persistent snapshots can be used to move freely between states with different transaction stacks. Terminating a transaction invalidates all transactions *and snapshots* that contains it in their transaction stack, weakening the persistence of snapshots inside transactions: snapshots are only persistent until one of their enclosing transactions is terminated.

It would be simpler to prevent the capture or restore of persistent snapshots as long as one transaction is active, that is, to allow transactions only at the “leaves” of the search tree. Our more flexible discipline allows many combinations that would be ruled out by such a restriction, in particular the two following important use-cases:

Arbitrary search in the current context. Within a transaction or any amount of nested transactions, it is possible to call a function that implements its own search sub-procedure using the full Store API (persistent or semi-persistent), without using any of the snapshots or transactions in the ambient context. The validity of the pre-existing snapshots and transactions is unchanged by this local search.

Moving to a different context and then coming back. At any point during a Store-using computation, it is possible to take a persistent snapshot S of the current store state, restore a different snapshot S' (an older state in the store history, or a sibling state), perform arbitrary store operations there, and restore S to continue the search as if nothing happened. This is valid as long as the operations performed outside S preserve the validity of the transactions in the stack of S .

4.4 Implementing Transactions

Transactions are implemented by adding a new kind of information in the graph, *transaction nodes*. Starting a transaction when the current root of the version tree is A (shown in Figure 5a) creates a new transaction node T_A that tracks the transaction (shown in Figure 5b). This does not affect the values of references: node T_A has the same mapping as node A .

When the transaction is rolled back, arbitrary nodes may have been added, as shown in Figure 5c. We remove the transaction node T_A from the graph – that is, we mark the node as *invalid*. We also remove (invalidate) all historic descendants of T_A , so in particular the correctness of the version tree is preserved. The initial state is restored: A becomes the current root again (Figure 5a). This is only valid if the current root of the version tree was “inside” the transaction, that is, if it is a node that is a current descendant of T_A . We keep track of that information in the transaction node (it is updated by *reroot*) and fail if the current root is not inside the transaction; otherwise, the transformation would end up with two root nodes in the version tree, the previous root and A .

“Removing” a node is implemented by marking it, or one of its current descendants, as *Invalid*. Which nodes to mark is an implementation detail; it suffices to mark enough nodes that, when called on an invalid snapshot or transaction, *restore*, *commit*, and *rollback* encounter an invalid node

and fail before they modify the current state. Our current implementation marks each transaction node – T_A and any child transaction – as well as the current root B .

Calling `restore` on a persistent snapshot must update the current state to apply the `Diff` nodes along the path, but also revert the edges of those `Diff` nodes and update their data to allow restoring in the other direction later. For transactions, `rollback` only updates the current state without touching the `Diff` nodes, leading to a small but measurable efficiency gain.

5 Testing and Benchmarks

5.1 Testing Store with Monolith

We used `Monolith` [Pottier, 2021], an OCaml testing framework that implements a specific form of state-based property-based testing called *model*-based testing. It takes a description of the API to be tested, a reference implementation (*model*) of the API, generates random sequences of API calls and checks that the real implementation matches the model.

To test `Store`, we wrote a reference implementation, designed to be as simple and clear as possible without any efficiency requirement; one could consider it an executable specification. The property we ask `Monolith` to check is that the real and reference implementations agree. The reference implementation represents functional mappings as a persistent map from unique integer indices (representing references). This is a homogeneous representation (all references must have the same value type) for simplicity: we only use integer values in tests. Each snapshot carries such a functional mapping, as well as a list of transactions that it depends on (as described in Section 4.3). A transaction is a snapshot, with a mutable boolean flag indicating whether it is still valid. Finally, a store is represented by a mutable reference to a snapshot; the active transactions are the transactions that the current snapshot depends on. The data definitions of our reference implementation are as follows:

```

type 'a sref = { key : int; default : 'a }
type 'a mapping = 'a Map.Make(Int).t

type 'a snapshot = { state : 'a mapping;
                    transactions : 'a transaction list; }
and 'a transaction = { snapshot : 'a snapshot;
                      mutable terminated : bool; }
and 'a store = 'a snapshot ref

```

We mention our testing approach explicitly because we have found it *unreasonably effective*. The fuzzer we get from `Monolith` behaves, in our experience, exactly like a correctness oracle. After any code change, you run the fuzzing test, and either it finds a bug in a few seconds or the code is correct. If it finds a bug, it starts looking for a smaller test sequence that also fails, and waiting for about 10 seconds will consistently produce a small, readable sequence of operations that can be replayed to understand what is going on.

Writing complex code with a correctness oracle at hand is a liberating experience. Wondering about why a particular line of code is necessary? Remove it, run the testsuite, and you see. Thinking of reordering two state changes and wondering if there is an interaction between them? Just try it.

We believe that model-based testing is unreasonably useful for `Store` because (1) we have a relatively small and simple API, so all interesting interactions are covered by random search and (2) we gave a lot of thought to expressing clear specifications, which in turn make it easy to write a precise reference implementation.

5.2 Microbenchmarks

We studied the performance of our Store library on synthetic microbenchmarks that let us simulate a variety of different usage scenarios. These benchmarks perform almost only operations on references, so they magnify the performance differences between implementations compared to real-world programs – where most of the time is typically spent elsewhere. We would typically consider overheads of up to 30% as small – unlikely to be noticeable in real-world programs, 2×-5× as moderate, and above 10× as large.

Our main goal is to establish that if users need *some* form of backtracking in a (possibly small) part of their program, using Store is always a good choice, they will not suffer a noticeable performance degradation compared to a library that supports fewer features, in particular compared to third-party libraries specialized for semi-persistence, and compared to built-in OCaml references when no backtracking at all is used. Before our work on Store, when François Pottier needed a Union-Find implementation with (non-nested) backtracking, he implemented the [union-find](#) library as a functor over a store-like interface, so that users that do not need backtracking do not pay a cost – they instantiate the functor with built-in references. We want to encourage users to drop this parametrization strategy and use Store unconditionally, by showing that Store has best-in-class performance for all relevant workloads.

Implementations. We compare the following implementations:

Store Our implementation.

Ref Native OCaml references; they do not support backtracking of any kind, and they are the gold standard for “raw” get/set operations.

TransactionalRef A “journaled” store by François Pottier, implemented in [union-find](#) for the needs of [Inferno](#), that only supports non-nested (semi-persistent) transactions.

BacktrackingRef An earlier “journaled” implementation of Store that we wrote, that only supports semi-persistence. A single dynamic array (the “log”) stores all antioperations, and ephemeral snapshots are denoted by positions inside this array. BacktrackingRef performs a record elision optimization.

Facile The backtrackable (semi-persistent) references of the [Facile](#) library, a well-established constraint-programming framework for OCaml, written with performance in mind.³

Facile uses a “journaled” implementation with record elision, similar to ours. (Record elision is easier to implement for semi-persistent implementations; our combination of persistent snapshots and record elision is the novelty.)

Colibri2 The original backtrackable (semi-persistent) references of the Colibri2 constraint-programming and SMT solver, written in OCaml. Colibri2 uses a “fat node” representation where the previous values of each reference are stored within the reference itself and the work of restoring an earlier version is done lazily on access, providing a constant-time rollback operation. This implementation has better memory-liveness properties, due to history being stored locally within each reference, but as evidenced by this section the on-demand approach has a noticeable overhead due to the extra check in the performance-critical get operation. We discussed this with the authors of Colibri2 who changed their implementation to be similar to ours in January 2024 (the numbers in this section correspond to the previous, distinctive implementation).

³Facile was written in 2005, and found to be comparable with state-of-the-art constraint solvers of the time: slower than Ilog Solver 4.3, faster than ECLiPSe 5.2.

Map An implementation using persistent maps (the Map module of the OCaml standard library): $O(\log n)$ get/set, but $O(1)$ capture/restore. This corresponds to the “full persistence” approach we mentioned in the introduction. We expect it to be quite slow due to the logarithmic factor.

Vector an implementation using dynamic arrays, provided by the [union-find](#) library, where backtracking operations copy the array. This corresponds to the “full copy” approach we mentioned in the introduction. It has fast get/set operations ($O(1)$), but very slow capture/restore operations ($O(n)$ in the number of references).

We expect Vector to be a solid baseline for the use-cases we had in mind when implementing Store – infrequent backtracking operations so get/set dominate performance.

Benchmarks. We consider the following synthetic benchmarks.

Raw creates 1024 references, then performs a series of 32 reads and 4 writes per reference in a loop repeated 1000 times.

Transactional (abort) creates 1024 references, then perform a series of reads and writes in a loop. Each iteration of the loop is performed in a failed (aborted) transaction.

We run the following variants, to simulate a variety of workloads:

get 128 reads per reference, no writes, 200 iterations

set few no reads, only 64 references are written to (once) in total, that is only $\frac{1}{16}$ of all references, 40000 iterations

set 1 no reads, each reference is written exactly once, 6400 iterations

set 16 no reads, each reference is written 16 times, 600 iterations

We also run the “set few” version in a successful (rather than failed) transaction with the same parameters, marked with “(commit)”.

Capture-heavy is the same as **Transactional**, but with different parameters to test the case where backtracking operations are much more frequent, with only a few reference accesses per transaction. We perform 16 writes and 64 reads per transaction in total, spread over 4 references in the “small” version (all references are touched in a single transaction) and 1024 references in the “large” version (most references are untouched in each transaction).

Backtracking is the same as **Raw**, except that each iteration of the loop starts a new *nested* transaction level. All transactions are failed (rolled back) once the loop completes. The loop is repeated 1000 times, which is also the nesting depth.

Results summary. The results of the microbenchmarks are summarized in Figure 6. The results are normalized relative to the Store implementation to show relative performance in the different tasks. The absolute benchmarks results are available in the appendices.

For reasons of space, we only provide a high-level summary of the results here. Detailed analyses of each benchmark are included in Appendix C.

Our general conclusion is that TransactionalRef, BacktrackingRef, Facile and Store are the best implementations, they perform very reliably over all benchmarks, with essentially no overhead over built-in references in the Raw benchmark. With the exception of the “set 1” variant where Vector shines, they are always the *best* implementations. For the benchmarks where they are supported they have very close performance.

BacktrackingRef and Facile are able to perform as well as TransactionalRef despite supporting nested transactions, and Store performs as well as those two despite supporting both persistent snapshots and semi-persistent transactions.

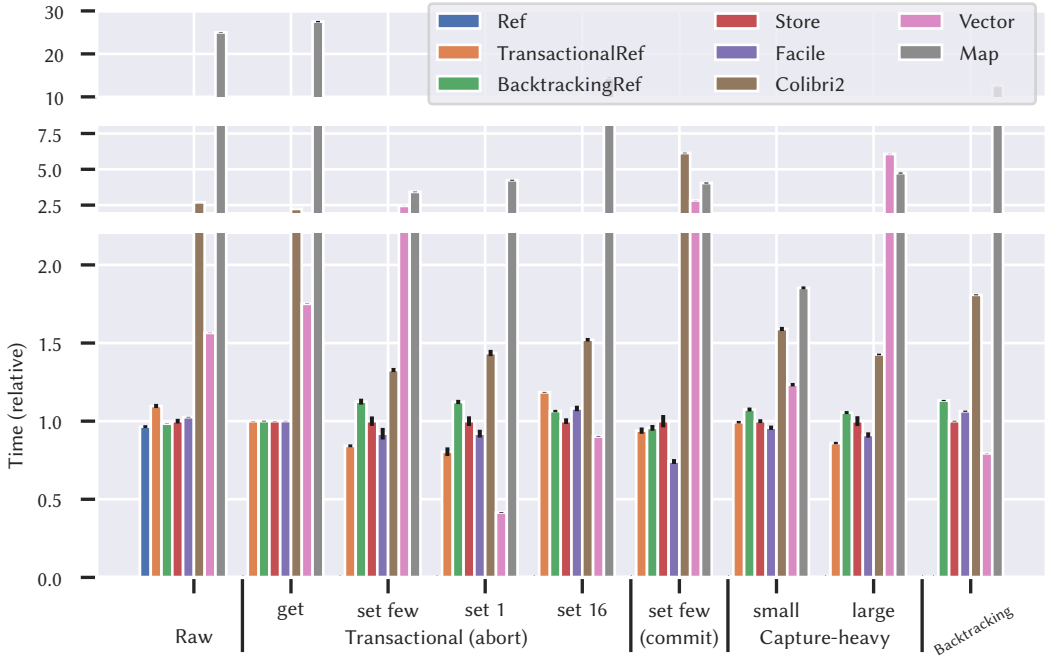


Fig. 6. Micro-benchmark results

Colibri2 is consistently slower than the other backtrackable implementations. This is due to the high runtime cost of its on-demand approach, incurring additional costs on each get and set operation.

This suggests that our objective for Store of always being a good choice – despite supporting more features – is reached. It also shows the advantage of providing snapshottable stores as an independent library that can be optimized once.

Note on API safety. The API provided by Store is very explicit not only about the store, but also about the state represented by backtracking levels (transaction objects). On the other hand, both Colibri2 and Facile use a mix of explicit (pop in Colibri2 and cut in Facile both take an explicit backtracking level) and implicit state (push implicitly creates a new backtracking level in Colibri2 and backtrack implicitly operates on the last non-cut backtracking level in Facile). This makes using their APIs outside of the solvers they were designed for error-prone; in fact, we initially used both APIs incorrectly when writing the benchmarks.

Details on Facile. Facile performs very well on the commit transactional workflow, because its implementation trades space for time, resulting in worse liveness properties. On commit, the diff nodes that allowed backtracking become useless, but Facile leaves them on its backtracking stack in a disabled (Cut) state. The time complexity of commit becomes linear in the number of checkpoints, rather than the number of modified references, but the space complexity becomes linear in the total number of modified references, rather than the references modified in the current history. On workflows that mix a few long-running snapshots with many short-running commit transaction, such as Inferno in the presence of backtracking, this choice can result in arbitrary increase in memory consumption. On a micro-benchmark (a variant of our Transactional-Commit

benchmark, with 10,000 rounds) simulating such a workflow we observe a memory consumption of 6Mo for Store and 645Mo for Facile.

Details on Vector. Vector performs surprisingly well, despite an extra indirection and bound checking. But it suffers from very bad behaviors on “large support” workloads, where only a few references are modified per transaction. Our “Capture-heavy (large)” test simulates them, and Vector is 6× slower than Store. We believe that this is the most common situation in real-world workloads, and have observed even worse behaviors, for example Vector is 52× slower than Store on one of our [Inferno](#) macro-benchmarks.

The best case for Vector is when each reference is modified exactly once per transaction. Indeed, all other implementations need to perform extra work on set that corresponds to a sort of per-reference copy-on-write; if we set all references after a snapshot, the total copy work should be at least as much as copying the vector on capture, with worse constant factors. We do observe excellent performance for Vector in the “set 1” variant of `Transactional`, which simulates this. But we do not know of programs in the wild with similar workloads.

If there are fewer references set per transaction, as in our “set few” variant, Vector is doing worse than journaled implementations. (Empirically we observed a break-even point on this benchmark when a fourth of the references are set per transaction.) On the other hand, when each reference is modified many times per transaction, as in the “set 16” variant, then journaled implementations benefit from record elision, reducing the advantage of Vector.

Details on Map. Map performs especially poorly in all benchmarks, except the capture-heavy benchmarks. This is expected since capture and restore have no cost for Map, but even in these favorable conditions Map is amongst the slowest implementations.

5.3 Macrobenchmarks

In order to validate the conclusions from microbenchmarks in more realistic scenarios, we adapted existing programs that perform some sort of backtracking, to use the Store interface. This gives a more realistic view of performance differences one can expect in practice. We detail the various macro-benchmarks in [Appendix B](#), with only a brief summary here.

[Inferno](#) re-checks the explicitly-typed programs elaborated by its type-inference engine. Our original use-case for Store was the introduction of GADTs, which requires undoing changes to a Union-Find of type equations.

[Figure 7a](#) measures type-checking a large explicitly-typed term that does not actually contain GADTs (the common case). Store is noticeably faster than Vector, the previous best choice.

[Figure 7b](#) measures type-checking a small explicitly-typed GADT example. Vector behaves terribly (this is a “large support” situation) and Store is much better than other choices.

[Figure 7c](#) measures [Inferno](#) type inference on a ML program. As mentioned earlier, [Inferno](#) uses (non-nested) transactions to roll back partial unifications in case of unification failure, and the `TransactionalRef` implementation of François Pottier was written specifically for this use-case. Our results show that Store can replace `TransactionalRef` for this use-case.

[Figure 7d](#) measures the performance of a constraint-based random generator of well-typed terms, which is an independent reimplementaion distinct from the [Inferno](#) codebase. The generator interleaves top-down program generation with constraint-solving, to quickly discard ill-typed terms and share type-checking work across similar terms. This benchmark uses the persistent Store API: the entire state of the generator (initially written without Store) is persistent, and was easy to integrate with the persistent Store API. In contrast, it would be fairly difficult to use the transactional API in a structured way.

	Time	Relative		Time	Relative		Time	Relative	
	Store	0.21s	1.0x	Store	0.02s	1x	T-Ref	0.03s	1x
	Vector	0.28s	1.3x	Map	0.08s	4x	Store	0.03s	1x
	Map	0.88s	4.2x	Vector	1.3s	70x	Map	0.09s	3x
(a) <i>Inferno</i> type checking (without GADTs)				(b) <i>Inferno</i> type checking (GADT example)			Vector	1.78s	52x
							(c) <i>Inferno</i> type inference (short transactions)		
Implementation	Time	Relative	Implementation	Time	Relative				
Store (persistent)	262ms	1.00	base (hand-optimized)	1.35s	1.00				
Map	334ms	1.27	Store	1.63s	1.20				
(d) Generator of well-typed terms			Store (persistent)	1.76s	1.30				
			Vector	4.03s	2.99				
						(e) Sudoku solver			

Fig. 7. Macro benchmarks

Finally, Figure 7e represents results on a backtracking-heavy program, an optimized Sudoku solver implemented in OCaml by Alain Frisch in 2005. The original implementation uses a hand-optimized “full copy” approach, taking a copy of the Sudoku board state on backtracking points. (Our test is on a 25×25 board.) Our results show that replacing the hand-optimized backtracking logic by Store only results in a 20% overhead, that using the persistent API instead is slightly slower, and that Vector would be much worse, $3\times$ slower than the original implementation.

6 Related Work

6.1 Snapshottable References

We searched the OCaml, Haskell, Scala and Rust ecosystems for previous libraries providing “snapshots as a service”, and were surprised not to find any.⁴ Some larger systems implement snapshottable references internally for their own purpose, in particular SAT/SMT solvers and constraint solvers; but they did not seem to consider releasing this as an independent library. In our experience, designing Store as an independent library led us to consider a variety of workloads more thoroughly, and improved our design and implementation.

Union-Find. The inspiration to think of “snapshottable store” as a library of its own came from the [union-find](#) OCaml library, which provides a Union-Find implementation parametrized over a “store”, a few simplistic store implementations, and the `StoreTransactionalRef` implementation supporting non-nested snapshots.

Coincidentally, the closest library we found to “snapshots as a service” is the Rust crate [ena](#), which implements a Union-Find data structure *and* provides an `undo_log` module offering a snapshot abstraction. This crate was extracted from the codebase of `rustc`, the Rust compiler, to be shared

⁴The [undo-redo](#) Rust crate is the closest we found. It keeps a history of “edit events” on some structure, and can call an “undo” callback associated to each event. It seems designed to record events at the scale of human interactions – human modifications to a document, etc. – rather than fine-grained changes, and would be fairly inefficient for our use-cases. It provides “record”, with a linear history (like most semi-persistent implementations) and “histories”, which allows a branching history with a git-like model of explicit branches.

with other Rust projects with a need for Union-Find. The implementation of `undo_log`⁵ provides a semi-persistent interface with a transactional flavor (`commit` and `rollback`), implemented with a global dynamic array of changes to undo. In particular, snapshots are not persistent, with dynamic checks and explicit panics if invalid snapshots are used. It implements the simplest form of record elision, which is to skip any logging when no valid snapshots exist.

`ena` supports arbitrary edit actions with undo callbacks (“custom operations”), but provides built-in support for creating and setting references. Those references are stored in a large dynamic array, with indices passed to the user. In consequence, a given undo log is parametrized over a fixed type of values, and references of different types cannot be combined in a single undo log – this makes using them more cumbersome for some applications, see our discussion of the Rust type-checker below. In contrast, our heterogeneous store can contain references of any type.

Search monads. If we cannot find “snapshots as a service”, we looked for such code bundled into a larger abstraction, namely a backtracking/search library. We have not found interesting code to snapshot state in search monads or logic programming monads.

Software Transactional Memory. Software Transactional Memory libraries are designed for concurrency rather than sequential use. In particular, their main concern is to detect races with another transaction running concurrently. STM libraries typically do implement a form of journaling, but with different requirements that make a comparison difficult. In particular, the implementations that we studied⁶ cannot implement record elision, as they need to track the previous *and* final value of each transaction variable – they cannot elide all tracking even if the variable was already modified by the continuation.

Bespoke implementations in types, solvers. We surveyed implementations of snapshottable stores hidden inside type checkers (we surveyed GHC, Scala 2 and 3, Rust, OCaml), SAT/SMT solvers (CVC5, Z3) and constraint solvers (`Facile`, `choco-solver`). For reasons of space, this content is moved in Appendix A.

6.2 Mutable and Persistent Interfaces

Our API provides a *mutable* interface: mutation operations modify the input store directly:

`update : store * params -> unit`. Another choice would be to provide a *persistent* interface, where mutation operations leave the input store unchanged, and return another store containing the modification. We write `pstore` to emphasize that the store is persistent:

```
val update : pstore * params -> pstore
```

Functional programming typically encourages persistent data structures, whose transparent referency helps for program reasoning. Using linear types (when provided by the source language) can provide similar benefits for mutable interfaces, reformulated using a linear function that consumes its input:

```
val update : store * params -> store
```

Conversely, the mutable (or linear) interface is often preferred for performance reasons. Some structures have efficient persistent implementations, but other structures have mutable versions with better complexity or noticeably lower constant factors. In the case of `Store`, the mutable API

⁵https://github.com/rust-lang/ena/blob/12584218/src/undo_log.rs

⁶We studied the OCaml library `keas` [Karvonen, 2024], the (C) STM implementation in the GHC runtime [Harris and Marlow, 2004], the Scala implementation in the Zio library [Goes, 2019], and the Go implementation `vMVCC` [Chang, 2023; Chang, Jung, Sharma, Tassarotti, Kaashoek, and Zeldovich, 2023].

makes snapshot capture explicit, instead of forcing the result of every update to be persistent, enabling record elision as a key optimization.

Some implementations expose a persistent interface only, but rely on reference-counting schemes to know when the input store is uniquely owned, and perform a mutable update in that case – they dynamically switch to the linear API. See for example Puente [2017], Stokke [2018], or the Functional but In-Place style popularized by Koka [Reinking, Xie, Moura, and Leijen, 2021]. This has the potential to be a “best of both worlds” solution, but only in systems where the cost of reference counting is already paid by the runtime or accepted as standard practice – it is a diffuse cost that must be paid by all users to enable this capability.

6.3 Transient Views of Persistent Data Structures

Some persistent data structures provide a *transient* view into the data structure, on which mutable updates can be applied imperatively, which can then be turned back into a persistent state:

```
val transient : pstate -> state      val mutably : (* higher-order combinator *)
val persistent : state -> pstate    pstate -> (state -> unit) -> pstate
```

The transient combinator can be used, for example, to efficiently add a lot of elements at once into a persistent collection. This is a pattern popularized by the Clojure community [Hickey and contributors, 2024], based on seminal ideas by Bagwell [2001]. Transient data structures can be found in many languages. For example, transient vectors and hash-maps can be found in Scala’s standard library, but also in the JavaScript library `immutable.js` [Byron, 2024], and in the Python library `persistent` [Gustafsson, 2023]. The C++ library `immer` [Puente, 2017] provides transients Relaxed Radix Balanced (RRB) vectors.

Our interface is the other way around: we expose the mutable API by default, but our snapshots are persistent, letting users capture persistent versions at points of interest in their code, typically around an operation they may want to backtrack over.

The two styles are equally expressive: we can implement a persistent store API with transient views, and conversely a mutable-with-snapshot API can be built on top of persistent-with-transient-views APIs. Our work focuses on enabling forms of persistence for data structures that are typically provided with a mutable API only, with an easy migration path for existing users.

6.4 State-of-the-Art Algorithms

Our work provides an *easy* way to equip an imperative data structure with backtracking – more generally, persistent snapshots. We of course do not expect the result to be competitive with specialized algorithms.

The standard complexity of a Union-Find implementation is $O(n\alpha(n))$ for a sequence of n union and find operations, with a $O(\log n / \log \log n)$ worst-case complexity for each operation in the sequence. If we require backtracking support (an operation to undo the last union operation), Westbrook and Tarjan [1989] prove a lower-bound of $\Omega(n \log n / \log \log n)$ for n operations, and Apostolico, Italiano, Gambosi, and Talamo [1994] provide an optimal implementation providing an $O(\log n / \log \log n)$ worst-case bound per union and find operation, with a total space cost of $O(n)$ for the whole sequence of operations. Their `backtrack : graph -> int -> unit` operation runs in time $O(1)$, and it is in fact able to undo the n most recent union operations.

We have not implemented this algorithm, nor are we aware of existing implementations, but our intuition is that this algorithm would have noticeably higher constant factors than the traditional Union-Find implementation. In contrast, our approach requires no new algorithmic expertise (except to implement our Store library once and for all), it provides a much worse complexity of $O(n)$ for the backtracking operation (that is infrequent in the workloads we are considering, relatively to

find and get queries), and very low constant factor overheads for existing operations – which are performance-critical for our workloads. Our space overhead is $O(n)$, as with state-of-the-art algorithms.

Demaine, Langerman, and Price [2008] present a persistent trie data structure, which is unrelated to our current interest, but it is of interest to us for two reasons. First, to our non-expert knowledge it presents a state-of-the-art implementation of persistent dynamic arrays (which can be resized dynamically), using a sophisticated “rebuilding” approach to interleave resizing work with updates – if you know of Okasaki’s technique to amortize the reversal of a list to implement a persistent queue, think of a much harder version of this idea. Second, it contains a very useful, detailed discussion of notions of persistence used in algorithmic research, which we tried to summarize in our introduction. Coming back to persistent (resizable) arrays: the standard approach for persistent arrays comes from Dietz [1989], where each access operation has cost $O(\log \log n)$ in expectation (it is randomized), where n is the total number of operations performed so far. This dependence on the number of operations is problematic for many use-cases, including ours – we only have such a dependence on backtrack operations, and want to avoid them on access operations. Demaine, Langerman, and Price [2008] lower it to $O(\log \log \Delta)$, where Δ is the total size of the array.

Driscoll, Sarnak, Sleator, and Tarjan [1989] expose generic techniques to add partial persistence and full persistence to existing data structures. These techniques are not encapsulated as libraries, they require changing the data structure and its operations in a systematic way. They apply to all data structures that can be seen as a graph of nodes with *bounded in-degree* – there is a global bound on the number of parents of each node. They are designed to provide $O(1)$ access to any version in the tree, and typically have higher constant factors than we would like. As it happens, the usual Union-Find data structure does not have bounded in-degree, as an arbitrary number of nodes can point to the same representative.

6.5 Static Checking and Formal Verification

Conchon and Filliâtre [2008] present a static checking discipline for semi-persistent data structures, based on ghost updates in Why3, a programming language designed for deductive verification. One could also use linear types or unique ownership to capture semi-persistence. Our OCaml implementation performs no static checking, but we invalidate our data structures at runtime in such a way that incorrect use results in a clear dynamic failure rather than unspecified behavior.

Conchon and Filliâtre [2007] propose persistent arrays and a persistent Union-Find library written in OCaml, and verify them in Coq. (The Union-Find implementation is built on the persistent arrays, so in particular it has bad liveness properties, it retains the memory of all nodes forever.) They use a shallow embedding of OCaml in Coq with an explicit heap, and express specifications using dependent types. This approach leads to verbose specifications. On the contrary, we benefit from Separation Logic and provide simpler specifications. Conchon and Filliâtre [2007] verify the termination of functions of the library, which we do not. We are confident that we can enhance our specifications and proofs with time credits [Charguéraud and Pottier, 2019] to verify both the termination and the time complexity of our implementation. Our proof does establish that the version graph remains acyclic, which is the key argument needed for termination.

Moine, Charguéraud, and Pottier [2022] propose the only formal verification of a transient data structure that we are aware of. They verify both functional correctness and time complexity of a transient stack in Separation Logic, using CFML [Charguéraud, 2022]. They represent the shared mutable state between snapshots using a dedicated assertion. Thanks to Iris support for monotone ghost state, we do not need such an assertion: our specifications are simpler.

7 Future Work

Verification. We verified the persistent core of Store, forcing us to build a precise model of the subtle implementation. In order of expected difficulty, next steps are first, to include complexity bounds in the specifications, and second, to extend the mechanized proofs to the semi-persistent API, which requires invalidating snapshots (and transactions).

Custom operations. Store currently supports a single mutable datatype, namely references. This is enough, as all mutable datatypes can be built on top of mutable references. For example, one can define a snapshottable dynamic array as a store reference over an array of store references, and build snapshottable hashtables on top of it.

We believe however that some datatypes would benefit performance-wise from being integrated more directly into our stores, by extending our version nodes with higher-level operations – adding a value to a dynamic array, writing a table at a given key, etc.

We are planning to use Store in the Alt-Ergo SMT solver [Bury, Clément, Coquereau, Conchon, Contejean, Olivera, El Hara, Iguernlala, Lescuyer, Mebsout, Roux, and Villemot, 2015], which would require support for custom operations.

One could of course hardcode such higher-level operations in the Store implementation (the backtrackable trail of Z3 is hardcoded in this way), but we would prefer to let users define “custom operations” following a certain abstract interface (the context-dependent objects of CVC5 provide this). We have started working on this abstract interface and played with several iterations of this idea; in particular, we believe that it is possible to combine custom operations with record elision. A difficulty is to find the right balance between generality and performance: some interfaces are more expressive than others, but they suffer from higher constant factors.

Confluence. Consider a user manipulating two snapshottable union-find graphs, each with its own store. They may decide to “merge” the graphs together – and start unifying nodes from both sides. We do not provide support for this. It is possible to just keep a product of stores, and restore/capture them together (rustc does this), but better support for this use-case could be useful in some scenarios – that we have not encountered yet.

Rebuilding. Journalled implementations, including Store, are optimized for “single-threaded” computations where switching from one snapshot to another is rare. Their performance breaks down if trying, for example, to evolve two different versions in lockstep. This is a limit to the generality of our implementation. Improving on this probably requires being able to track several copies of the “global state” simultaneously. For example, one could ask to *rebuild* a given snapshot, a costly operation that would turn it into an independent copy of the state – in particular, its validity would not depend on active transactions anymore.

The algorithmics literature studies how to perform this rebuilding implicitly, whenever edit chains become long enough that it is worth it – see in particular Chuang [1994, 1992]. This introduces other costs, in particular in space, and makes it harder for users to reason about performance. We would rather keep this an explicit operation.

Our current implementation choice, where each reference really has a unique field storing its current state – instead of being an index into a copiable structure – is in tension with rebuilding, we do not see how to do it. It seems challenging to offer this capability without hurting constant factors and/or our memory-liveness properties (Section 2.5).

Acknowledgments

François Pottier implemented [unionFind](#), a Union-Find library parametrized over a notion of store, which was used to provide persistent and ephemeral store, and a version providing non-nested backtracking. This library is used in *Inferno*, a type-inference library used for research prototypes.

Gabriel Scherer worked with Olivier Martinot to implement GADTs in *Inferno*, which brought the need for nested backtracking as a new “store” instance for `unionFind`. This motivated the first, semi-persistent version of *Store*, written by Gabriel Scherer and described in [Scherer \[2023\]](#). Providing a persistent implementation while keeping record elision was noted as an interesting open problem. Jean-Christophe Filliâtre and François Pottier provided interesting feedback on the first iteration of this work, along with anonymous reviewers who pushed us to write benchmarks.

Basile Clément got interested in *Store* from previous experience with backtracking structures as the current maintainer of the SMT Solver *Alt-Ergo*. He realized that the implementation technique of Baker (see Section 2.1), popularized in the OCaml community by [Conchon and Filliâtre \[2007\]](#), could be used to provide a persistent implementation for store. Basile Clément wrote a second, better, persistent implementation, which then grew up to become the version described in this paper. He also implemented its [Monolith](#) model (see Section 5.1) which later proved invaluable in evolving both the API and the implementation. Gabriel Scherer and Basile Clément worked on cleaning up the implementation, understanding and resolving advanced questions – in particular the meaning of generations, and what kind of combinations of the persistent and semi-persistent APIs could be allowed, documentation (with help from Guillaume Bury), a detailed documentation of the implementation (essentially an informal correctness proof as OCaml comments). A first version of the micro-benchmarks was written by Gabriel Scherer, but the vast majority of the work was done afterwards by Basile Clément, as repeated cycles of analysis of the results, improvements and bias-fixing. The macro-benchmarks were done by Gabriel Scherer in the process of preparing the present article.

The *Store* implementation raises an interesting question for program verification – a mutable data-structure exposing a persistent model. Clément Allain and Alexandre Moine joined the project with their Coq+Iris expertise. Alexandre Moine verified the persistent core of *Store*, without generations and record elision. Clément Allain wrote a second iteration with a finer-grained model that also covers generations and record elision. (We have not attempted to formalize the transactional/semi-persistent part of *Store*.)

The Coq section of the paper was written by Alexandre Moine with help from Clément Allain; Basile Clément wrote the micro-benchmark section; the rest is mostly from Gabriel Scherer.

After submission, we sent a draft version of this article to François Pottier who provided excellent feedback – together with our anonymous reviewers. Shortly after, François Pottier proposed a different Coq presentation of the *Store* API, where ownership of the reference values in the current version is carried by special points-to predicates, instead of being centralized in a global map. This is a scientific contribution of independent interest, but unfortunately we received it too late to include it in the current publication. We hope that it gets its own exposition, possibly in the context of a follow-up on future work.

A Bespoke Implementations in Types, Solvers

We surveyed implementations of snapshottable stores hidden inside type checkers (we surveyed GHC, Scala 2 and 3, Rust, OCaml), SAT/SMT solvers (CVC5, Z3) and constraint solvers ([Facile](#), [choco-solver](#)).

Type checkers. The GHC type-checker does not implement backtracking of any form.

The Scala 2 type-checker implements journaled backtracking for its type inference variables, a simple semi-persistent implementation with a global list of undo actions.⁷ No record elision. Interestingly, another custom undo log is maintained in the function inliner – the project could benefit from generic snapshotability support.

The Scala 3 type-checker implements a snapshot/restore interface for the entire type-checking state⁸, but the snapshot logic is intentionally trivial as all this state is maintained in fully persistent data structures. (Looking for use-cases of the snapshot function shows all the places where the type-checker resorts to backtracking.)

The Rust type-checker implements “undo logs” for its mutable state, using the `undo_log` module of the `ena` crate we mentioned earlier. Because undo logs are homogeneous, different components of the type-checking state are stored in different undo logs. A module in the type-checker gathers all these logs⁹, with a single function to snapshot and restore them all at once.

The OCaml type-checker implements a snapshotability mechanism for its type variables, whose implementation is also inspired by (or a rediscovery of) Baker.¹⁰ The implementation seems to support full persistence, but it seems that it is only used in a semi-persistent way in the compiler codebase. This implementation performs a simplified form of record elision, based on the birth date of the reference rather than the timestamp or generation of its last write. Indeed, each type variable has a unique identifier implemented as consecutive integers starting at 0, which can also serve as a “birth date” for the type variable. The snapshot implementation tracks the value of the type identifier counter when the last snapshot was taken. When performing a write on a type, it performs record elision if the type has a higher identifier than the last snapshot – it was created after the snapshot was taken. This heuristic is less precise than our record elision, but it comes for free once type identifiers are there. It seems fairly effective for a type-checker due to a sort of generational phenomenon: most type variables are modified a lot shortly after they are created, and more rarely afterward. (Disabling this form of elision makes type-checking about 5% slower on some files of the compiler codebase.)

Constraint solvers and SAT/SMT solvers. Based on discussions with implementors of automated theorem projects, we conjecture that all SMT solvers include some version of a general snapshottable store – but of course they did not tell anyone until we explicitly asked them. The only explicit mention we found is in the recent overview paper on CVC5 [Barbosa, Barrett, Brain, Kremer, Lachnitt, Mann, Mohamed, Mohamed, Niemetz, Nötzli, Ozdemir, Preiner, Reynolds, Sheng, Tinelli, and Zohar, 2022], which describes “Context-Dependent Data Structures” (Section 2.4)¹¹, and currently supports context-dependent maybe/option values, append-only lists, dequeues, insert-only hashsets, and hashmaps. Z3 simply adds support for adding arbitrary edit events on the “trail”, and does not seem to support record elision.¹² The implementations in SMT solvers are semi-persistent, and their API is influenced by the internal vocabulary of SAT search algorithms; typically, one does not backtrack to a given snapshot, but to a “decision level”.

Constraint-based solvers seem to also implement semi-persistent snapshottable structures, and we have found implementations of record elision, which is relatively natural in the semi-persistent

⁷<https://github.com/scala/scala/blob/2429854/src/reflect/scala/reflect/internal/tpe/TypeConstraints.scala#L26-L76>

⁸<https://github.com/scala/scala3/blob/0e36424/compiler/src/dotty/tools/dotc/core/TypeState.scala#L29-L43>

⁹https://github.com/rust-lang/rust/blob/9afdb8d1/compiler/rustc_infer/src/infer/undo_log.rs#L19-L32

¹⁰<https://github.com/ocaml/ocaml/blob/572aeb5f/typing/types.ml#L490-L514>, <https://github.com/ocaml/ocaml/blob/572aeb5f/typing/types.ml#L851-L874>

¹¹<https://github.com/cvc5/cvc5/blob/92caabc7/src/context/context.h>

¹²<https://github.com/Z3Prover/z3/blob/2880ea39/src/util/trail.h>

case. We mentioned `Facile`, an OCaml implementation, but for example the Java constraint solver `choco-solver` also has support for generic “trails”, and performs record elision¹³.

B Macrobenchmarks Details

This appendix contains the full details on the macrobenchmarks mentioned in Section 5.3.

B.1 System F Type-Checking in <https://gitlab.inria.fr/fpottier/infernoInferno>

The `Inferno` project implements type-inference for a small ML language, and for well-typed terms it produces a “witness” or an “elaboration”, which is an explicitly-typed version of the input program in a variant of System F. `Inferno` includes a type-checker for this explicitly language, which is much simpler than type inference and can be used to catch bugs in the type inference machinery.

This explicit type checker uses a Union-Find data structure to check equality between types. We worked on a prototype extension of `Inferno` with GADTs, which required to add backtracking to the Union-Find graph of System F types to support local type-equality assumptions that are undone when leaving the scope of a GADT equation.

This was our initial motivation for implementing `Store`, and an ideal scenario for journaled implementations. `Vector` is a bad choice because we are in the “large support” worst-case: most backtracking points (that is, pattern-matching clauses containing GADTs) are short-lived and modify only a few Union-Find nodes. On the other hand, `Map` introduces an important overhead, even when the code does not use GADTs.

Now that we have `Store` implemented we can replace `Vector` with it and compare performance. We use `Inferno`’s own performance test, which is to generate a large *random* term (with a generator design to produce well-typed terms), infer its type and check its explicitly-typed version.

The results in Figure 7a show that in this real program performing many other operations than `Store` operations, using `Vector` is 1.3× slower than using our `Store` implementation, and using `Map` is 4.2× slower. Adopting `Store` is easy and comes with a direct, noticeable performance improvement.

The large random term type-checked in the test above does *not* contain any GADTs¹⁴ (the random generator does not know about them), so no snapshots are actually taken when running this test. This is a best case for `Vector` – it does not suffer from the “large support” situation.

We do not have good, representative test programs that contain a reasonable frequency of GADT constructs, but as a limit case we checked the performance of the type-checker on a small GADT example – a very short program that *only* checks GADT features, checked 1000 times in a loop. The results (below) should be taken with a grain of salt, as this is closer to microbenchmark territory again. For this limit test shown in Figure 7b, the System F type-checker remains 4× slower with `Map` than with `Store`, but using `Vector` now performs terribly, almost 70× slower, due to the “large support” situation.

B.2 System F Type Inference with GADTs (`Inferno`)

The previous test measures the performance of type-checking of explicitly-typed terms in `Inferno`. `Inferno` also uses a Union-Find data structure during inference of ML terms, performing inference via unification as usual. As we explained previously, `Inferno` implements a transactional behavior for unification of types: a single unification constraint is decomposed in many variable-variable

¹³<https://github.com/chocoteam/choco-solver/blob/efb697ea/solver/src/main/java/org/chocosolver/memory/trailing/EnvironmentTrailing.java>, <https://github.com/chocoteam/choco-solver/blob/efb697ea/solver/src/main/java/org/chocosolver/memory/trailing/StoredInt.java#L33-L48>

¹⁴The implementation of the type-checker must support GADTs, and thus use a snapshottable store. For this specific benchmark without GADTs, we tried using built-in references out of curiosity, and the performance is the same as `Store`.

unifications, but if any of those fail, we revert all changes to the inference state caused by this unification constraint in order to generate clear error messages. We measure the type-inference work for (again) a large randomly-generated ML term, with our Union-Find graph instantiated by different store implementations.

This workload has a relatively high number of backtracking points, most of which perform little work (most type-type unification are on small types that perform few variable-variable unifications). This workload is a worst-case scenario for full-copy implementations such as Vector, but it is a best case for full-persistence implementations such as Map. There are no nested transactions, so François Pottier’s TransactionalRef implementation can be used – in fact, it was designed precisely for this use-case, so it is the gold standard for this test.

We see in Figure 7c that Store has the same performance as TransactionalRef despite being much more general; Map is much slower, and Vector is unacceptably slow.

B.3 Random Generation of Well-Typed Programs

This macrobenchmark comes from an independent research project, also related to type-inference but with a codebase separate from *Inferno*. The program is a research prototype exploring the use of constraint-based type inference for random generation of well-typed programs. The overall idea is that instead of writing random generators that contain typing-checking logic to guarantee well-typedness, we can combine a type-agnostic program generator and a random-generation-agnostic typechecker by interleaving random generation steps with constraint-solving step. For more details, see a preliminary technical report at Scherer [2024], which links to our prototype implementation.

In this program, the entire state of the constraint solver is persistent. We can express this using our persistent snapshots. In contrast, it would be fairly difficult to use the semi-persistent API in a structured way, because the random generation is provided by a search monad using a standard interface, which does not provide callback points for backtracking operations. This example validates the expressivity benefits of the persistent API.

The workload contains a very high number of backtracking points (one per term-former in the AST of the generated program), with relatively few operations on the solver state in-between. This is a best case for a full-persistence implementation such as Map, but we see in Figure 7d that using Store instead still provides clear performance benefits: we measured a 27% performance improvement on a specific run of the random term generator.

B.4 Sudoku Solver

We wanted to test backtracking programs that are not doing type-checking of any form. We are interested in using Store in SAT or SMT context, but SAT/SMT engines have deeply ingrained forms of backtracking and it is not so easy to port existing solvers to Store. Instead we looked for Sudoku solvers written as constraint-solving programs, which are typically simpler. We found an OCaml implementation of a Sudoku solver¹⁵ written by Alain Frisch in 2005 with performance in mind, and we adapted it to use Store.

A constraint-based Sudoku solver operates on a “board state”, which tracks the possible values (the “domain”) of each board position. Whenever the domain of a board position is refined, we propagate constraints to other positions whose domain could be refined in turn (in the same row, column or block). Once all constraints have been propagated fully, we have to perform backtracking: choose a yet-undetermined position, and try each of the possible value of its domain – backtracking any state change after each attempt fails. Sudoku solvers must represent the board state efficiently

¹⁵<http://alain.frisch.fr/sudoku.html>

(this solver uses an array of integers, where integers are used as bitsets to represent the domains), propagate constraints efficiently, and use good heuristics to decide which position to backtrack on.

Alain Frisch’s Sudoku solver uses a hand-crafted “full copy” implementation, that copies the full board state at each backtracking point. The implementation is careful about reusing buffers to avoid allocations when possible. The state is fixed and relatively small, so copy is cheap – we used a test benchmark on a 25×25 sudoku board, so the state is an array of 625 integers.

base is Alain Frisch’s hand-crafted implementation, and it remains the fastest. Store adds 20% overhead. Store (persistent) uses our persistent API rather than our semi-persistent API; it performs slightly worse at 30% overhead. Finally, Vector is 3× slower. Vector is noticeably slower because it induces a memory representation that is less compact than the hand-written implementation¹⁶ and cannot reuse buffers.

Our conclusion is that even though Store does not beat a hand-crafted full-copy implementation of backtracking in this case, its low overhead remains acceptable on backtracking-intensive programs. Using Store instead of carefully copying temporary buffers may be a good deal for some programmers.

C Detailed Microbenchmarks Results and Analyses

We introduce our microbenchmarks in Section 5.2, but for reasons of space we only gave a high-level summary of the results. The current appendix contains more details on our benchmarking setup, the results of each benchmark, and a summary analysis of the results.

C.1 Methodology

Performing accurate microbenchmarks is very difficult.

We account for runtime noise by running benchmarks many times, and can provide intervals / error estimates (we use the `hyperfine` tool). All the micro benchmarks are run on a machine with an AMD Ryzen Threadripper 3990X processor and 264Go of RAM. Hyper-threading and frequency scaling are disabled, the frequency is set to its minimum of 2.2GHz, and the benchmarks are run sequentially on a single isolated core, so that the noise level of running the same binary repeatedly is very low.

Other sources of measurement biases are harder to detect and control. Our general approach is to ensure that we know how to explain the benchmark results, and carefully study each result that we do not understand – more often than not, this comes from a measurement bias that must be fixed to give accurate results. For example, we found performance swings of up to 10% due to code alignment effects. (We now run our benchmarks with 16 different alignments to control this.)

In our opinion, the main threat to validity of the results below is that we have had access one noise-controlled benchmarking machine with a specific AMD ThreadRipper processor, and that some of the fine-grained qualitative comparisons may be different on other processors or architectures. This is an issue with microbenchmarks, which give a very detailed view of performance but are more sensitive to system differences. The macrobenchmark discussed in Section 5.3 are more robust in that regard.

C.2 Benchmark Parameters

All benchmarks are purely synthetic, and they are parametrized by the following environment variables.

¹⁶To measure the importance of the compact memory implementation, we replaced the `int` array implementation of Alain Frisch by an exactly equivalent `int ref` array implementation, introducing one indirection in the memory represent. This introduces a 48% overhead, larger than Store.

ROUNDS the benchmark does *something* in a loop, ROUNDS time; the total time should scale linearly with this variable (but this may not be just a **for** loop, there may be an environment growing from one round to the next).

NCREATE, NREAD, NWRITE : the logarithm of the number of references to create,read,write each round.

We use three sets of parameters to simulate different workloads:

default represents our default workload where backtracking operations are rare, and reads dominate writes. We use NCREATE=10, NREAD=16, NWRITE=12, with $4 \cdot 1024$ writes and $32 \cdot 1024$ reads per transaction. All references are touched in each transaction, so this is an ideal case for Vector.

capture-heavy tests a limit case where backtracking operations are much more frequent, with only a handful of get/set calls per transaction. We use NCREATE=2, NWRITE=4, NREAD=6, with 16 writes and 64 reads per transaction (spread over 4 references).

capture-heavy-large-support is a variant of capture-heavy where there are many references around, but only a few of them are touched by each transaction. We use NCREATE=10, NWRITE=4, NREAD=6, with 16 writes and 64 reads per transaction (spread over 1024 references; most references are untouched at each round).

All benchmarks where it is applicable were run in an “abort” configuration (each round is within a failed transaction that must be rolled back) and a “commit” configuration (each round is within a successful transaction that is not rolled back).

C.3 Per-Benchmark Results and Analysis

C.3.1 Raw.

Mixed get/set workload	Time (ms)	Relative
Ref	102.9 ± 0.98	1.00 ± 0.01
BacktrackingRef	104.9 ± 0.11	1.02 ± 0.01
Store	106.6 ± 1.59	1.04 ± 0.02
Facile	109.3 ± 0.21	1.06 ± 0.01
TransactionalRef	117.0 ± 1.43	1.14 ± 0.02
Vector	166.8 ± 0.10	1.62 ± 0.02
Colibri2	287.2 ± 0.15	2.79 ± 0.03
Map	2664.4 ± 6.79	25.90 ± 0.26

Ref is the gold standard for this benchmark. Store, BacktrackingRef and Facile have a small overhead (around 5%). TransactionalRef is a bit slower (around 15% overhead): it performs two writes per set instead of one. Vector is even slower (60% overhead), probably due to additional indirections and bound checks. Colibri2 is almost 3 times slower due to the extra checks on each read. Map is an order of magnitude slower than the rest (25-27 times).

TransactionalRef has a slower set operation in the absence of backtracking (two polymorphic writes instead of one), but it keeps the same code in the presence of backtracking (thanks to its restriction to non-nested transactions). It will perform better (relatively to BacktrackingRef, Facile, Store) in the Transactional benchmarks that follow.

C.3.2 *Transactional, `abort-Only*. In a transactional scenario, Ref cannot be used. Store, TransactionalRef, BacktrackingRef and Facile are the fastest and all within 5% of each other; Vector is about 40% slower, Colibri2 is about 2× slower and Map is 22× slower.

	Time (ms)	Relative
Mixed get/set workload		
Store	92.9 ± 0.38	1.00 ± 0.01
Facile	93.9 ± 0.15	1.01 ± 0.00
TransactionalRef	94.3 ± 0.65	1.02 ± 0.01
BacktrackingRef	96.1 ± 0.66	1.03 ± 0.01
Vector	128.4 ± 0.11	1.38 ± 0.01
Colibri2	197.7 ± 0.48	2.13 ± 0.01
Map	2014.1 ± 5.32	21.69 ± 0.11

Get-only workload. Vector is 70% slower than the other implementations on the “get” variant, partially due to performing many unnecessary copies.

	Time (ms)	Relative
Get		
Store	97.5 ± 0.11	1.00 ± 0.00
TransactionalRef	97.5 ± 0.10	1.00 ± 0.00
BacktrackingRef	97.5 ± 0.22	1.00 ± 0.00
Facile	97.5 ± 0.10	1.00 ± 0.00
Vector	170.6 ± 0.11	1.75 ± 0.00
Colibri2	217.6 ± 0.11	2.23 ± 0.00
Map	2680.7 ± 15.91	27.51 ± 0.17

Varying ratios of set. For set-only benchmarks we measure three different ratios of write: in “set few”, only one out of 16 references is modified (once) at each round. In “set 1”, each reference is modified exactly once. In “set 16”, each reference is modified 16 times.

	Time (ms)	Relative
Set few		
TransactionalRef	58.1 ± 0.65	1.00 ± 0.02
Facile	63.4 ± 2.55	1.09 ± 0.05
Store	69.0 ± 2.08	1.19 ± 0.04
BacktrackingRef	77.6 ± 1.31	1.34 ± 0.03
Colibri2	91.5 ± 0.91	1.58 ± 0.02
Vector	168.7 ± 0.29	2.91 ± 0.03
Map	236.0 ± 1.34	4.06 ± 0.05

	Time (ms)	Relative
Set 1		
Vector	70.1 ± 0.42	1.00 ± 0.01
TransactionalRef	135.7 ± 4.68	1.94 ± 0.07
Facile	155.2 ± 4.21	2.21 ± 0.06
Store	168.7 ± 5.23	2.41 ± 0.08
BacktrackingRef	189.4 ± 2.29	2.70 ± 0.04
Colibri2	242.3 ± 3.38	3.46 ± 0.05
Map	714.9 ± 5.88	10.20 ± 0.10

	Time (ms)	Relative
Set 16		
Vector	70.7 ± 0.15	1.00 ± 0.00
Store	78.4 ± 1.42	1.11 ± 0.02
BacktrackingRef	83.5 ± 0.57	1.18 ± 0.01
Facile	84.8 ± 1.37	1.20 ± 0.02
TransactionalRef	92.9 ± 0.18	1.31 ± 0.00
Colibri2	119.3 ± 0.91	1.69 ± 0.01
Map	1133.8 ± 7.29	16.04 ± 0.11

Vector shines on the “set 1” variant where it is $2\times$ faster than other implementations. The “set 1” variant is the best-case scenario for the “full copy” approach, since all other implementations degrade to also doing a full copy with worse constant factors. This advantage goes away if many set operations are performed in a transaction and record elision kicks in: in the “16 set” variant, Vector is only about 10-15% faster than the other implementations. It also goes away if only a subset of the references are modified: in the “set few” variant, it is $3\times$ slower than the best implementation.¹⁷

The relative performance of TransactionalRef can be explained by its set implementation: while its *elided* write is slower than the other journaled implementations, its *non-elided* write is simpler due to not supporting nesting. This gives it a performance boost in scenarios that do not allow record elision (the “set 1” variant and the “large” capture-heavy variant); that goes away as the number of writes per reference increases (in the “16 set” variant and “small” capture-heavy variant).

More generally, the difference in performance between the journaled implementations boils down to relative efficiency of elided and non-elided writes. The default and set 16 configurations compare write performance, and the set and set few configurations compare non-elided write performance. TransactionalRef has a single write implementation that is faster than non-elided writes of other implementations but slower than their elided writes. Facile has fast non-elided writes, but slow elided writes. Store has fast elided writes, but slow non-elided writes (with an extra `caml_modify` compared to Facile). BacktrackingRef has slow elided and non-elided writes.

Colibri2 is generally slow, partially due to get operations being slower but also set operations are slower in general.

¹⁷In this particular benchmark, we find that the break-even point is when around one-fourth of the references are modified per transaction.

Capture-heavy variants. The Map implementation has a much smaller overhead in the “small” capture-heavy variant; however, even in this ideal scenario (few references and few read/write operations per transaction), it is still twice as slow as the journaled implementations. When the number of references increases, the logarithmic overhead shows up, as in the “large” capture-heavy variant – where Vector also performs much worse.

	Time (ms)	Relative
Capture-heavy, small support		
Facile	45.5 ± 0.49	1.00 ± 0.02
Store	48.9 ± 1.07	1.07 ± 0.03
TransactionalRef	50.2 ± 0.40	1.10 ± 0.01
BacktrackingRef	51.8 ± 0.87	1.14 ± 0.02
Vector	62.7 ± 0.38	1.38 ± 0.02
Map	101.9 ± 0.37	2.24 ± 0.03
Colibri2	160.4 ± 1.11	3.52 ± 0.04
Capture-heavy, large support		
TransactionalRef	80.6 ± 0.72	1.00 ± 0.01
Facile	85.4 ± 1.59	1.06 ± 0.02
Store	93.7 ± 2.96	1.16 ± 0.04
BacktrackingRef	98.8 ± 0.90	1.23 ± 0.02
Colibri2	133.7 ± 0.50	1.66 ± 0.02
Map	443.9 ± 3.73	5.51 ± 0.07
Vector	568.8 ± 0.75	7.06 ± 0.06

C.4 Transactional, `commit-Only

The results for `commit-only` transactional benchmarks are similar for most implementations, except for Colibri2 which does not support efficient `commit` operations. The `commit` operation for Store is marginally slower than for the other implementations, but still competitive.

Facile has a very efficient `commit` operation, which allows it to shine on these microbenchmarks. This is due to an optimization made by Facile when calling `commit` on the first transaction in the stack, so that no transactions remain after the call to `commit`: instead of walking back the transaction stack, Facile throws it away entirely. This advantage goes away for nested transactions.

	Time (ms)	Relative
Mixed get/set workload		
Facile	89.7 ± 0.53	1.00 ± 0.01
Store	91.8 ± 0.44	1.02 ± 0.01
TransactionalRef	93.9 ± 0.19	1.05 ± 0.01
BacktrackingRef	94.2 ± 0.88	1.05 ± 0.01
Vector	128.5 ± 0.07	1.43 ± 0.01
Colibri2	298.6 ± 0.61	3.33 ± 0.02

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	Time (ms)	Relative
Mixed get/set workload		
Map	1996.5 ± 10.18	22.27 ± 0.17

	Time (ms)	Relative
Get		
Store	97.4 ± 0.22	1.00 ± 0.00
Facile	97.5 ± 0.09	1.00 ± 0.00
BacktrackingRef	97.5 ± 0.23	1.00 ± 0.00
TransactionalRef	97.5 ± 0.10	1.00 ± 0.00
Vector	170.6 ± 0.10	1.75 ± 0.00
Colibri2	219.1 ± 0.16	2.25 ± 0.01
Map	2398.4 ± 11.95	24.62 ± 0.13

	Time (ms)	Relative
Set few		
Facile	43.7 ± 0.94	1.00 ± 0.03
TransactionalRef	55.3 ± 1.18	1.27 ± 0.04
BacktrackingRef	56.3 ± 1.09	1.29 ± 0.04
Store	58.9 ± 2.31	1.35 ± 0.06
Vector	166.4 ± 0.31	3.81 ± 0.08
Map	238.0 ± 2.28	5.45 ± 0.13
Colibri2	361.3 ± 1.35	8.28 ± 0.18

	Time (ms)	Relative
Set 1		
Vector	69.4 ± 0.11	1.00 ± 0.00
Facile	111.4 ± 2.15	1.61 ± 0.03
TransactionalRef	135.4 ± 4.40	1.95 ± 0.06
Store	151.8 ± 0.63	2.19 ± 0.01
BacktrackingRef	154.7 ± 3.08	2.23 ± 0.04
Map	808.5 ± 6.12	11.65 ± 0.09
Colibri2	925.4 ± 3.59	13.34 ± 0.06

	Time (ms)	Relative
Set 16		
Vector	70.7 ± 0.25	1.00 ± 0.01
Store	77.2 ± 0.37	1.09 ± 0.01
BacktrackingRef	80.3 ± 0.31	1.14 ± 0.01

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	Time (ms)	Relative
Set 16		
Facile	80.6 ± 0.44	1.14 ± 0.01
TransactionalRef	92.7 ± 0.22	1.31 ± 0.01
Colibri2	183.8 ± 2.39	2.60 ± 0.04
Map	1168.2 ± 8.07	16.53 ± 0.13

	Time (ms)	Relative
Capture-heavy, small support		
Facile	45.5 ± 0.49	1.00 ± 0.02
Store	48.9 ± 1.07	1.07 ± 0.03
TransactionalRef	50.2 ± 0.40	1.10 ± 0.01
BacktrackingRef	51.8 ± 0.87	1.14 ± 0.02
Vector	62.7 ± 0.38	1.38 ± 0.02
Map	101.9 ± 0.37	2.24 ± 0.03
Colibri2	160.4 ± 1.11	3.52 ± 0.04

	Time (ms)	Relative
Capture-heavy, large support		
Facile	60.4 ± 1.10	1.00 ± 0.03
TransactionalRef	71.0 ± 0.40	1.18 ± 0.02
Store	72.9 ± 1.71	1.21 ± 0.04
BacktrackingRef	73.0 ± 0.93	1.21 ± 0.03
Colibri2	386.0 ± 1.59	6.39 ± 0.12
Map	437.1 ± 7.06	7.23 ± 0.18
Vector	561.3 ± 1.94	9.29 ± 0.17

C.4.1 Backtracking.

	Time (ms)	Relative
Implementation		
Vector	216.9 ± 0.21	1.00 ± 0.00
Store	273.3 ± 0.65	1.26 ± 0.00
Facile	290.7 ± 1.11	1.34 ± 0.01
BacktrackingRef	309.5 ± 1.04	1.43 ± 0.01
Colibri2	494.3 ± 0.91	2.28 ± 0.00
Map	3464.2 ± 5.18	15.97 ± 0.03

This benchmark tests deeply nested backtracking chains, with our standard set parameters where all references are set 4 times and read 16 time in each round. This scenario is again favorable to our full-copy baseline Vector, with “journalled” implementations being somewhat slower at 29%-43% overhead. Map remains very slow, 16× slower than Vector. (TransactionalRef does not support nested transactions, so it cannot be used here.)

C.4.2 Persistent API. Finally, we use the Backtracking benchmark, which performs deeply nested backtracking, to measure the performance difference between the persistent and semi-persistent operations of Store – we run the same workload with the tentatively function reimplemented on top of capture/restore. On this test, we observe a 50% overhead for the persistent API.

Implementation	Time (ms)	Relative
Backtracking-abort	275.9 ± 0.74	1.00 ± 0.00
Backtracking-persistent	413.6 ± 3.19	1.50 ± 0.01

Remark. We conclude that there are *some* workloads where the semi-persistent API provides a noticeable performance difference. The difference, however, remains fairly small for a microbenchmark, and would typically not be noticeable for many end-user applications.

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