1 Introduction

Unboxing a single-constructor datatype. ML-family languages support both type abbreviations, which provide a synonym for an existing type, and datatypes (sums/variants and records) that provide a new type (distinct from previous types) specified by its constructors or fields.

Since 4.06 (June 2016), OCaml additionally supports unboxed datatypes, which are single-constructor variants or single-field records that behave like datatypes during type-checking (they are distinct types) and abbreviations at runtime – constructor application or pattern-matching are erased.

Unboxing individual constructors within a datatype. In ocaml/rfcs#14 Jeremy Yallop proposed to allow unboxing a constructor even if the datatype has other constructors:

```ml
type a_datatype = Short of int | Long of int list
```

(SML uses an explicit datatype keyword for datatype declarations, and Haskell uses data for datatypes, type for abbreviations and newtype for unboxed datatypes.)

Unfolding without loops? To compute the head shape of a type expression `int t`, we need to inspect/unfold the definition of the datatype ` типа t`, which in turn requires computing the head shape of the parameters of its unboxed constructors. For example, with

```ml
type 'a t =
  | Foo of foo [@unboxed]
  | Bar of bar [@unboxed]
```

the head shape is the union of the shapes of foo and bar, whose computation may in turn require following (data)type definitions.

In presence of mutually-recursive datatypes, repeated unfolding may lead to non-termination. For a pathological example:

```ml
type loop = Loop of loop [@unboxed]
```

How can we compute type properties (in our case, head shape) by repeatedly unfolding datatype definitions without risking non-termination on cyclic definitions? This question is the topic of the present workshop submission.

Head tags and head shapes. We define the head tag `h` of a value to be either some (non-unboxed) datatype constructor `C` occurring at the head of the value, or a datatype constructor among a fixed set of primitive constructors `τ` with distinguished representations (`int`, `string`, `array`, `tuple`, `function`, etc.), or the worst approximation `⊥` which contains all possible values. We want to compute a head shape `H` for any type expression `r`, which is just a list of possible head approximations.

```latex
\begin{align*}
  h & := C | \tau | \top \\
  H & := \emptyset | H, h
\end{align*}
```

If we know when two head tags have disjoint low-level representations, and we know how to compute the head shape of a type expression, we can easily check the disjointness condition for unboxed constructors `C^{unboxed}` of `τ`: the unboxing annotation is valid if the tags in the head shape are pairwise disjoint.

In this document we do not discuss the first question (what is the low-level representation of each head tag and which definition of disjointness we use), which are low-level details related to the OCaml implementation. We only discuss how to compute those implementation-independent head shapes.
2 Type unfolding with dynamic cycle detection

Static or dynamic cycle detection? The problem of cycles also occur with type synonyms/abbreviations. OCaml will forbid cyclic type abbreviations such as

type 'a foo = ('a * 'a bar) and bar = 'a foo

This check is defined as a static check of well-formedness: the graph of dependencies from one abbreviation to another (the definition of foo mentions bar) must be acyclic. Note that cyclic mentions in datatypes are allowed, for example:

type 'a foo = Cons of ('a * 'a bar)

and bar = 'a foo

We could follow the same approach, by considering that, unlike constructors, unboxed constructors create dependencies in this sense: a cycle of reference must go through at least one boxed constructor to be accepted. However, we found that this approach is too restrictive in practice. For example:

type 'a thunk = unit -> 'a

type 'a stream =
| Next of ('a * 'a stream) thunk [@unboxed]
| End

The static discipline would consider that 'a stream depends on itself (in an unboxed position) due to the occurrence of 'a stream within an argument of thunk. But if we were to unfold the definition of thunk, we would notice that this recursive occurrence is under a function type, whose primitive tag function does not depend on its input or output types.

The dynamic detection mechanism we propose is more fine-grained than the static check, and in particular accepts this declaration.

Naive dynamic cycle detection. A natural idea when performing a series of unfoldings to compute a head shape is to remember the set of type definitions that we have already expanded. If we encounter a type definition that is already in this visited set, we are at risk of circularity and abort the computation, rejecting the definition.

However, this approach is disappointing in practice. Consider for example:

type 'a id = Id of 'a [@unboxed]

type t = Foo of int id id [@unboxed]

Computing the head shape of t would unfold id once, then in turn compute the shape of int id, and abort as id was already visited.

Call stacks. The two occurrences of id here are distinct and both occur in t. The second occurrence should thus count as a (second) dependency of t on id, not a use of id within itself!

Our solution is to track, for each type subexpression of our input, the definition in which it occurred (int id occurs in t). More generally, we track the path of unfoldings that led to this definition, which we call a call stack for this type expression. In this example, computing the shape of t (in the empty call stack) amounts to computing the shape of int id id in the call stack [t], remembering that these expressions come from the definition t, which we can write (((int[t] id)[t] id)[t]. Unfolding this definition brings us to the definition of id, so we now consider the expression (int[t] id)[t] (the parameter of the first id) in the call stack [t, id]. To compute this shape, we unfold the remaining id, but from (int[t] id)[t] we know that it comes from the call stack [t]; unfolding id again in this stack is not cyclic, we get int[t], and terminate with the primitive tag int.

Note: the name call stack comes from viewing datatype definitions as (mutually recursive) functions, and the problem of head shape computation as the evaluation of a call to one of these recursive functions. Our call stacks really correspond call stacks for those functions, in the setting of call-by-name evaluation where the argument of a function is not computed until needed — but its call stack comes from its application site.

3 Our algorithm

Let us define a toy grammar for types $\tau$ and datatype declarations $d$.

\[
\begin{align*}
\tau &::= a \mid (\tau_i) t \\
d &::= \text{type } (a_i)^t t = (C_j \text{ of } \tau_j)^t \left(\text{boxed of } \tau_k\right)^k
\end{align*}
\]

Each declaration comes with a family of (boxed) constructors and a family of unboxed constructors, either of which could be empty.

Our algorithm tracks the call stack in which type subexpressions appear. We represent this with annotated types $(\tau @ l)$, which contain a call stack $l$ at the top, and also on each subexpression.

\[
\begin{align*}
\tau @ l &::= a @ l \mid ((\tau_i @ l_i)^t) @ l \\
l &::= \emptyset \mid l, t
\end{align*}
\]

Head shape of a type declaration $d$.

\[
\text{Typedecl} \quad \begin{aligned}
\{((a_i @ \emptyset)^t) t) @ \emptyset &\Rightarrow R \\
(\text{type } (a_i)^t t = \ldots) &\Rightarrow R
\end{aligned}
\]

To compute the head shape of a type declaration type $(a_i)^t t=\ldots$, we simply compute the head shape of the type expression $(a_i)^t t$, with all type subexpressions annotated with the empty call stack $\emptyset$. This type declaration will be rejected by our implementation exactly if the computation returns CycIe, or if two head tags with non-disjoint representations are found in the result.
We can now define our algorithm as a judgment $\tau @ l \Rightarrow R$, which takes an annotated type expression $\tau @ l$ and returns a result $R$, either a head shape or a Cycle error.

$$R := H \mid \text{Cycle}$$

$$\prod \alpha l \Rightarrow \tau$$

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<tr>
<td><strong>Cycle</strong></td>
<td>$t \in l$</td>
<td>$((\alpha l)_i^t) @ l \Rightarrow \text{Cycle}$</td>
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<tr>
<td><strong>Prim</strong></td>
<td>$(t, \tilde{t}) \in \tilde{T}$</td>
<td>$((\alpha l)_i^t) @ l \Rightarrow \tilde{t}$</td>
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When computing the shape of the datatype $(\alpha l)_i^t$, the datatype parameters $\alpha_l$ could get instantiated with types of any shape. Our rule **Var** thus gives those type variables the shape $\tau$.

Before unfolding a type constructor, we check that it is not already in our call stack. If it is, the rule **Cycle** aborts with a Cycle result.

We assume a relation $\tilde{T}$ from certain datatype constructors to primitive shapes, used in the rule **Prim**.

The rule **Type** performs the unfolding of a datatype definition. Our input is a datatype $(\tau_i @ l)_i^t$ at some call stack $l$. We lookup the definition of the datatype $(\alpha l)_i^t$ in the global datatype definition environment, split it into a family of boxed constructors and a family of unboxed constructors. The resulting shape is obtained by concatenation of the tags of boxed constructors, and shapes of unboxed constructor arguments: for each boxed constructor $C_j$ of $\tau_i$ we add the tag $C_j$, and for each unboxed constructor $C_k^{\text{unboxed}}$ of $\tau_i$ we add the shape of $\tau_i^k$. There are two subtleties in the rule:

- Computing the shape of a $\tau_i$ may fail with a Cycle error; we use a merge $(\ldots, \text{Cycle}, \ldots)$ operator that propagates this error, or concatenates the result shapes.
- We define it below.

The $\tau_i^k$ mention the formal datatype parameters $(\alpha l)_i^t$ of the datatype declaration. We substitute them with the actual datatypes parameters $(\tau_i)_i^t$ used the type expression at hand. More precisely, we have annotated types $(\tau_i @ l)_i^t$, which we substitute under a non-annotated type $\tau_i^k$. To get an annotated type as expected, we need to use a call stack for all type subexpressions of $\tau_i^k$ that are not variables (each variable $\alpha_l$ gets the call stack $l$ from our input). Those subexpressions get the call stack $l$, $t$, tracing the fact that they come from the expansion of $t$ in the current context $l$.

To summarize, our substitution operation $\tau [\sigma]@[l]$ takes an unannotated type $\tau$, a substitution $\sigma$ from its type parameters to annotated types $\tau_i @ l$, and an “ambiant call stack” $l$ to use on all non-variable type expressions. We define it below.

$$\text{merge}(\ldots, \text{Cycle}, \ldots) = \text{Cycle}$$

$$\text{merge}((H_i)^j) = (H_i)^j$$

$$\alpha[\sigma]@[l] = \sigma(\alpha)$$

$$(\tau_i)^l t[\sigma]@[l] = ((\tau_i)[\sigma]@[l]) t @ l$$

### 3.1 Termination

We have a proof sketch (not a complete proof yet) of termination that goes as follow.

We say that the parent of a non-empty call stack $l$, $t$ is the call stack $l$.

We have to prove that any potentially-infinite derivation of $d \Rightarrow R$ is finite. It suffices to prove that any path of applications of the rule **Type** within such a derivation is finite. Indeed, the three other rules, **Var**, **Cycle** and **Prim**, terminate the derivation, and the width of each **Type** application is bounded by the maximal number of constructors of a type declaration in the global environment (which we assumed finite).

For a path $P$ of **Type** applications we look at the input types $(\tau_p @ l_p)^{p \in P}$ along this path, and in particular the path of locations $(l_p)^{p \in P}$. (We order $P$ by position in the derivation, with the smallest element $p_0$ corresponding to the root of the derivation.)

**Lemma.** If the stack $l_p$ occurs at position $p$ in the stack, all prefixes of $l$ can be found at some earlier position $p' \leq p$.

We build a tree structure on this path $P$. Note: the tree is not the derivation, is is overlayed over one linear path.

The root of the tree is the first element $p_0$ of the path. Remark that it is the unique element with the empty call stack $\emptyset$.

Any other element of the path has a call stack $l$, $t$; we place it as a children of the closest element earlier in the trace with the parent call stack $l$, which must exist by the lemma above. By construction, the parent of a child in the tree has the parent call stack.

Our termination argument is that this tree is finite (so the trace is finite):

- Its height is bounded: a call stack $l$ can only mention each datatype once, so the height of any call stack (and thus of the tree) is bounded by the number of datatypes in the global environment.
- Each node has a finite number of children. Intuitively the argument is if a node corresponds to some type $(\tau_i)^l t$, each child corresponds to one sub-expression occurring in the expansion of the datatype $t$, with each sub-expression occurring at most once among the children. (This is the delicate part of the proof.)