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Jan 2016 - Jul 2017: Northeastern University

Sep 2012 - Dec 2015: Gallium (INRIA Rocq.)

Formally improving the programming experience

So far:

- ① Implementation and research on OCaml
- ② Type-directed program inference
- ③ Program equivalence and canonical representations

Project:

- ① Canonical representations at higher types
- ② Tools with program equivalence
- ③ Multi-language programming systems

Integration: Parsifal

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Integration: Parsifal

Result: deciding equivalence

# Setting

We have tools to check that a program verifies a specification.

Few tools to check program equivalence.

Potential applications: verification of refactoring, consistency checking, program synthesis...

Pure functional programming: rich equivalences.

More useful, but more complex.

Fundamental challenge: equivalence is not well-understood.

# Equivalence in the full simply-typed $\lambda$ -calculus is decidable

“Deciding equivalence with sums and the empty type”

Gabriel Scherer

POPL 2017

<https://arxiv.org/abs/1610.01213>

# History

Simple types: formal model of **datatypes** in programming.

Decidability of equivalence:

- $\Lambda C(\alpha, \rightarrow)$ : Tait, 1967 or earlier.
- $\Lambda C(\alpha, \rightarrow, \times)$ : essentially the same proof.
- $\Lambda C(\alpha, \rightarrow, \times, 1)$ : essentially the same proof.
  
- $\Lambda C(\alpha, \rightarrow, \times, 1, +)$ : Ghani, 1995; Altenkirch, Dybjer, Hoffman, Scott: 2001; Balat, Di Cosmo, Fiore: 2004; Lindley, 2007; Ahmad, Licata, Harper, 2010.
  
- $\Lambda C(\alpha, \rightarrow, \times, 1, +, 0)$ : this work.

Open problem despite work: need a different approach.

```
module type PARAM = sig
  type error
  val process : input -> (output + error)
  ...
end

module Action (P : PARAM) = struct
  let process_or_stdout input =
    match process input with
    |  $\sigma_1$  out -> out
    |  $\sigma_2$  err -> report_error_stdout (); exit 1
  let process_or_email input =
    match process input with
    |  $\sigma_1$  out -> out
    |  $\sigma_2$  err -> report_error_email (); exit 2
  ...
end
```



## Intuition

0 represents impossible cases.

$$\frac{\Gamma \vdash t : 0 \quad \Gamma \vdash u_1, u_2 : A}{\Gamma \vdash u_1 \approx_{\eta} u_2 : A}$$

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## Question

What is a **canonical form** for equivalence of simply-typed terms?

Redundancy: two (syntactically) distinct terms that are equivalent.

Canonical representation: a syntax of programs with no redundancy:

$$(\approx_{\text{stx}}) \implies (\approx_{\text{sem}})$$

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With only functions and pairs, there is a reasonable notion of  $\beta$ -short  $\eta$ -long normal form. It does not scale to sums.

# Idea

Curry-Howard, again: programs as proofs.

The structure of

canonical forms

corresponds to the structure of

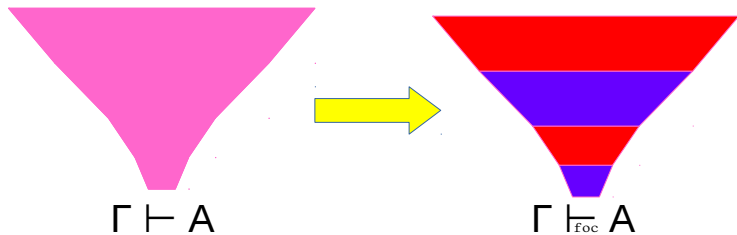
proof **search**

Restricting the search space restricts expression redundancy.

Research transfer from proof theory.

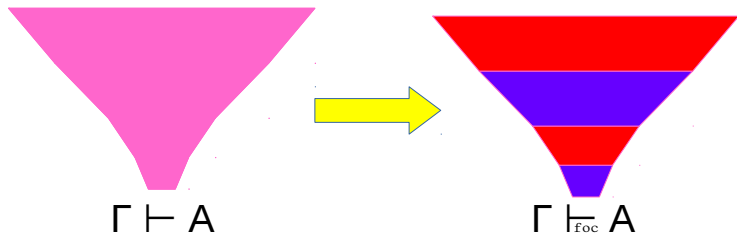
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(existing work)



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Gives a term representation ( $\vdash_{\text{foc}}$ ).

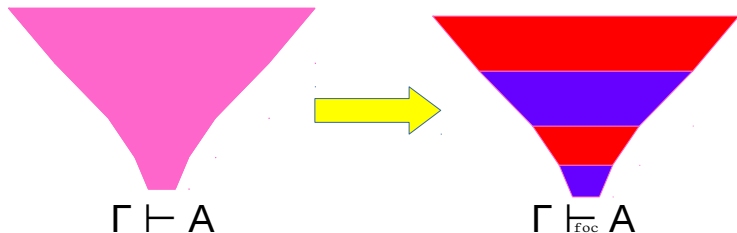
Not yet canonical.

And it preserves computational content!



# Proof search: Focusing

(existing work)



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Not yet canonical.

And it preserves computational content!

$$\Gamma \vdash t : A \quad \xrightarrow{\text{(new)}} \quad \exists v \approx_{\beta\eta} t, \quad \Gamma \vdash_{\text{foc}} v : A$$

## Proof search: Saturation

(my contribution).

Non-invertible steps: either  $(p : P)$  (value) or  $(\text{let } x = n[y : M] \text{ in } \dots)$  (environment).

Idea: make all possible deductions from the environment first.

Canonical representation, (locally) complete.

## Proof search: Saturation

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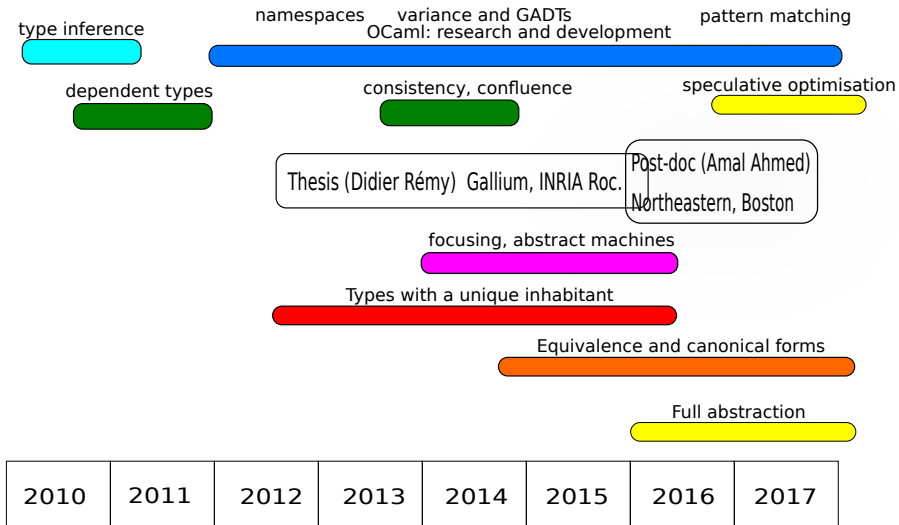
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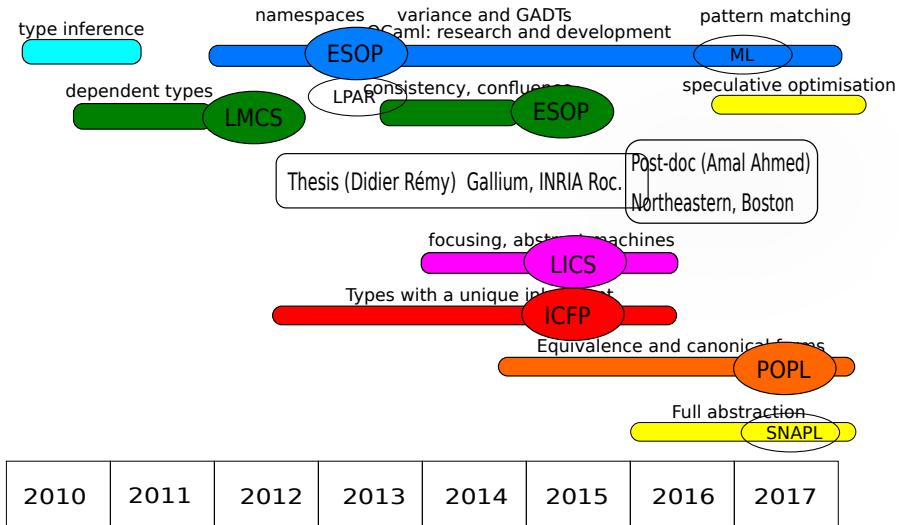
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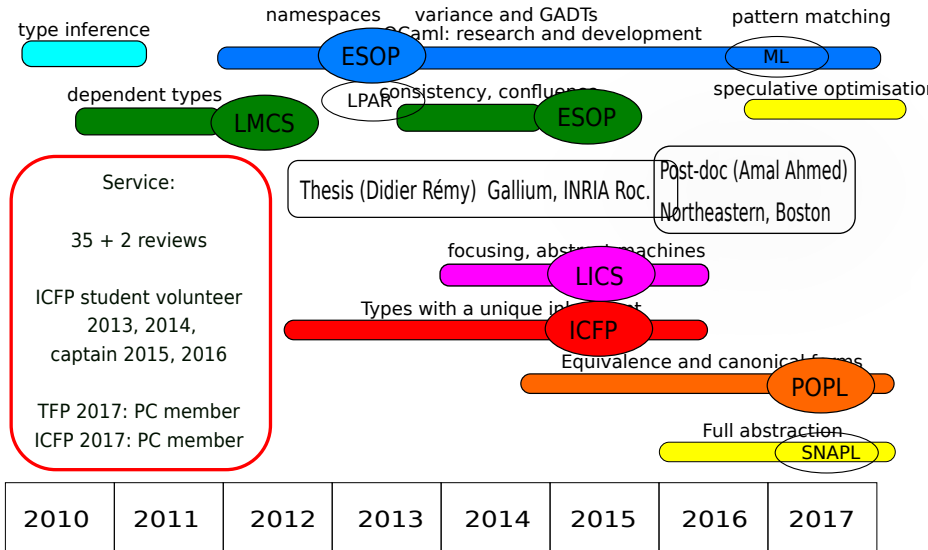
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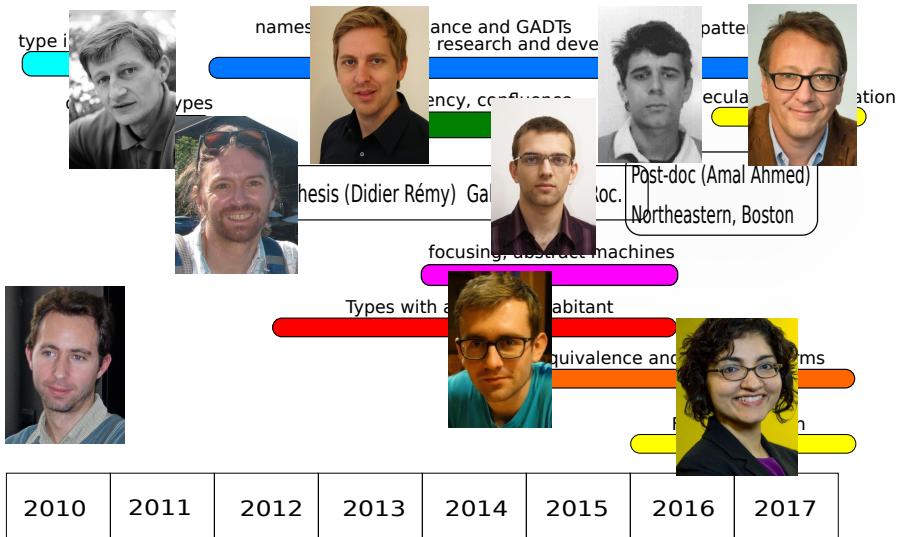
$$\frac{\Gamma \vdash t : 0 \quad \Gamma \vdash u_1, u_2 : A}{\Gamma \vdash u_1 \approx_{\eta} u_2 : A}$$

Saturation discovers  $t$ .









## 2012-2017: Research and development on OCaml

- technical contributions to the implementation (committer #2)
- community building: opening the development process (github, code reviews, social events)  
20 contributors in 2012, 93 in 2017
- research problems identified and studied

Example: ambiguous pattern variables, with Luc Maranget

- bug report from the Why3 team
- research and publication – ML workshop post-proceedings
- patch to the compiler, merged in 4.04.0
- cross-language discussions with Haskell, Rust designers

Community recognition:

PC member for the OCaml Workshop 2016, PC chair for 2017.



Theory, design and implementation of programming languages.

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Project: multi-language programming systems

# The Ultimate Language may not exist

Ideal (general-purpose) language design:  
simplicity/power compromise using powerful, orthogonal concepts.

More and more problem domains for general-purpose languages:  
distributed programming, web/mobile development...

Languages of today tend to evolve into behemoths by piling features up:  
C++, Scala, GHC Haskell, OCaml...

Does managing this complexity require super-human feats?

## Multi-language systems

**Proposal:** Multi-language programming systems.

Several smaller languages working together to cover the feature space.

(simpler?)

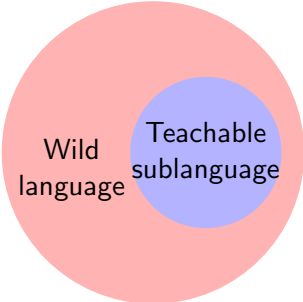
(Done in practice, but no design guarantees.)

To manage complexity, one should be able to **ignore** some languages of the system – and not pay for it.

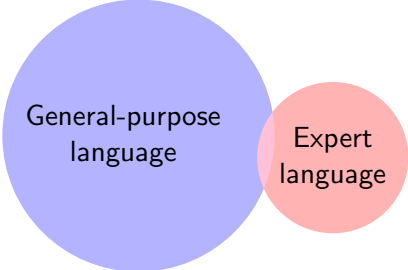
Multi-language system require specific **design** for graceful interoperation.

We must learn how to achieve this, rigorously.

# Multi-language stories



Abstraction leak?



Graceful interoperation?

# Full abstraction

(existing work)

$\llbracket - \rrbracket : S \longrightarrow T$  fully abstract:

$$a \approx b \implies \llbracket a \rrbracket \approx \llbracket b \rrbracket$$

Full abstraction preserves (equational) reasoning.

(Program equivalence again)

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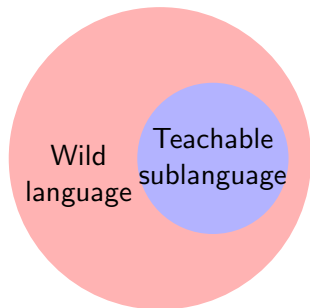
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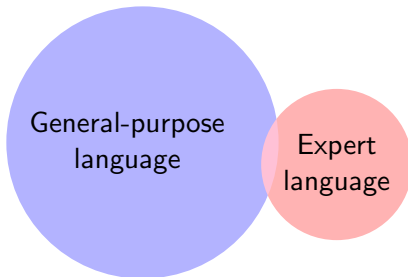
(new) **Claim:** full abstraction can be used to **formally** capture the **usability** properties of multi-language design.



# Full abstraction for multi-language systems

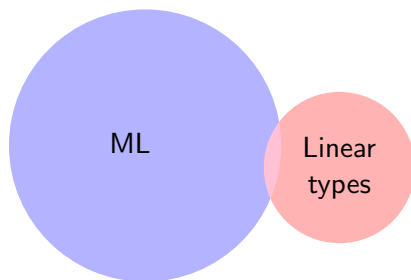


No abstraction leaks:  $T \xrightarrow{f.a.} W$



Graceful interoperation:  $G \xrightarrow{f.a.} (G + E)$   
(not symmetric)

## First instance in the works



Expert linear language allows safe lower-level programming.  
Efficiency and safety complement.

Other potential cases: Coq+OCaml, Why3+ML, safe FFI,  
interaction between proof assistants (Coq, Agda, Abella, Dedukti)...

## Challenges

Full abstraction not yet well-understood. Theoretical advances required.  
(simply-typed with recursion  $\rightarrow$  untyped: was POPL 2016 article)

How to weaken full-abstraction when it cannot hold?

Can this scale to full-fledged  $n$ -languages designs?

- G.S. and Amal Ahmed. "Search for Program Structure". **SNAPL**. 2017.
- G.S. "Deciding equivalence with sums and the empty type". **POPL**. 2017.
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