ArtiCheck: well-typed generic fuzzing for module interfaces

Thomas Braibant  Jonathan Protzenko  Gabriel Scherer
INRIA
http://gallium.inria.fr/blog/

Abstract
In spite of recent advances in program certification, testing remains a widely-used component of the software development cycle. Various flavors of testing exist: popular ones include unit testing, which consists in manually crafting test cases for specific parts of the code base, as well as quickcheck-style testing, where instances of a type are automatically generated to serve as test inputs.

These classical methods of testing can be thought of as internal testing: the test routines access the internal representation of the data structures to be checked. We propose a new method of external testing where the library only deals with a module interface. The data structures are exported as abstract types; the testing framework behaves like regular client code and combines functions exported by the module to build new elements of the various abstract types. Any counter-examples are then presented to the user.

Categories and Subject Descriptors D.2.5 [Software Engineering]: Testing and Debugging

Keywords  functional programming, testing, quickcheck

1. Introduction
Software development is hard. Industry practices still rely, for the better part, on tests to ensure the functional correctness of programs. Even in more sophisticated circles, such as the programming language research community, not everyone has switched to writing all their programs in Coq. Testing is thus a cornerstone of the development cycle. Moreover, even if the end goal is to fully certify a program using a proof assistant, it is still worthwhile to eliminate bugs early by running a cheap, efficient test framework.

Testing boils down to two different processes: generating test cases for test suites; then verifying that user-written assertions and specifications of program parts are not falsified by the test suites.

QuickCheck is a popular, efficient tool for that purpose. First, it provides a combinator library based on type-classes to build test case generators. Second, it provides a principled way for users to specify properties over functions. For instance, users may write predicates such as “reverse is an involution”. Then, the QuickCheck framework is able to create instances of the type being tested, e.g. lists of integers. The predicate is tested against these test cases, and any counter-example is reported to the user.

Our novel approach is motivated by some limitations of the QuickCheck framework. First, data-structures often have internal invariants that would not be broken when using the API of the data-structure. Thus, testing one particular function of such an API requires generating test-cases that meet these invariants. Yet, writing good generators for (involved) data-structures is tedious, or even plain hard. Consider the simple problem of generating binary search trees (BST), for instance. Being smarter than merely generating trees and filtering out those which are not search trees requires reimplementing a series of insertions and deletions into BSTs. But these functions are certainly already part of the code that is tested!

We argue that this low-level manipulation could be taken care of by the testing framework. That is, we argue that the framework should be able, by itself, to combine functions exported by the module we wish to test in order to build instances of the data-types defined in the same module. If the module exports the properties and the invariants that should hold, then the testing framework needs not see the implementation. In a nutshell, we want to move towards an external testing of abstract modules.

In the present document, we describe a library that does precisely that, dubbed ArtiCheck. The library is written in OCaml. While QuickCheck uses type-classes as a host language feature to conduct value generation, we show how to implement the proof search in library code – while remaining both type-safe and generic over the tested interfaces, thanks to GADTs.

2. The essence of external testing
In the present section, we illustrate the essential idea of external testing through a simple example, which is that of a module SIList whose type t represents sorted integer lists. The invariant is maintained by making t abstract and requiring the user to go through the exported functions empty and add.

This section, unfolding from the initial example, introduces the key ideas of external testing: a GADT type that describes well-typed applications in the simply-typed lambda calculus; a description of module signatures that we wish to test; type descriptors that record all the instances of a type that we managed to construct.

The point of view adopted in this section is intendedly simplistic. The design, as presented here, contains several obvious shortcomings. It allows, nonetheless, for a high-level overview of our principles, and paves the way for a more thorough discussion of our design (§3).

Here is the signature for our module of sorted integer lists. The sorted function represents the invariant that the module intends to preserve. The module admits a straightforward implementation, which we also show.

```ocaml
module SIList: sig
  type t
  val empty: t
  val add: t -> int -> t
  val sorted: t -> bool
```

[Copyright notice will appear here once 'preprint' option is removed.]
Roughly speaking, our goal is to generate, as if we were client code of the module, instances of type \( \tau \) using only the functions exported by the module. Therefore, one of our first requirements is a data structure for keeping track of the \( \tau \)'s created so far. We also need to keep track of the integers we have generated so far, since they are necessary to call the add function: ArtiCheck will thus manipulate several \( \text{ty} \)'s for all the types it handles.

The present section reviews the issues with the current design and incrementally addresses them.

### 3.1 A better algebra of types

Type descriptions only model function types so far. We want to support products and sums, to be able to generate them when they appear in function arguments, and to use them to discover new values when they are returned as function results.

One of the authors naively suggested that the data type be extended with cases for products and sums, such as:

- Prod: ('a,'c) fn * ('b,'c) fn -> ('a * 'b,'c) fn

It turns out that the constructor above does not model product arguments. If \( \text{a} \) is \( \text{int} \) -> \( \text{int} \) and \( \text{b} \) is \( \text{int} \) -> \( \text{float} \), not only do the \( \text{c} \) parameters fail to match, but the \( \text{a} * \text{b} \) parameter in the conclusion represents a pair of functions, rather than a function that returns a pair! Another snag is that the type of \( \text{eval} \) makes no sense in the case of a product. If the first parameter of type \( \text{a} * \text{b} \) fn represents a way to obtain a \( \text{b} \) from the product \( \text{a} \), then what use is the second parameter of \( \text{eval} \)?

Datatypes and function types are used in fundamentally different ways by the generator, which suggests splitting the \( \text{fn} \) type into two distinct GADTs -- inspired by the focusing literature [9]. The GADT \( \text{a} * \text{b} \) negative (neg for short) represents a computation of type \( \text{a} \) that produces a result of type \( \text{b} \). The GADT \( \text{a} \) positive (pos for short) represents a value, that is, the result of a computation.
We provide an injection from positive to negative types via the Ret constructor: a value of type 'a is also a constant computation.

We do not provide an injection from negative to positive types, which would be necessary to model nested arrows, that is, higher-order functions. One can take the example of the map function, which has type ("a -> 'b") -> 'a list -> 'b list: we explicitly disallow representing the 'a -> 'b part as a Fun constructor, as it would require us to synthesize instances of a function type (see §6 for a discussion). Note that the user can still use the Ty constructor to represent 'a -> 'b as an atomic type, initialized with its own test functions.

Our GADT does not accurately model tagged, n-ary sums of OCaml, nor records with named fields. We thus add a last Bij case; it allows the user to provide a two-way mapping between a built-in type (say, 'a option) and its ArtiCheck representation ('a, 'a). That way, ArtiCheck can work with regular OCaml data types by converting them back-and-forth. This change of representation incurs some changes on our evaluation functions as well. The eval function is split into several parts, which we detail right below.

Finally, the apply function, just like before, takes a computation along with a matching description, and generates a set of b. However, it now relies on product to exhaustively exhibit all possible arguments one can pass to the function.

We are now able to accurately model a calculus rich enough to test realistic signatures involving records, option types, and various ways to create functions.

3.2 Efficient construction of a set of instances

The (assuredly naive) scenario above reveals several pain points with the current design.

• We represent our sets using lists. We could use a more efficient data structure.

• If some function takes, say, a tuple, the code as it stands will construct the set of all possible tuples, map the function over the set, then finally call destruct on each resulting element to collect instances. Naturally, memory explosion ensues. We propose a symbolic algebra for sets of instances that mirrors the structure of positive types and avoids the need for holding all possible combinations in memory at the same time.

• A seemingly trivial optimization sends us off the rails by generating an insane number of instances. We explain how to optimize further the code while still retaining a well-behaved generation.

Sets of instances The first, natural optimization that comes to mind consists in dropping lists in favor of a more sophisticated data type. For reasons that will become obvious in the following, we chose to replace lists with arbitrary containers that have the following (object-like) type:

```ocaml
type 'a bag = {
  insert : 'a -> 'a bag;
  fold : 'b . ('a -> 'b -> 'b) -> 'b -> 'b;
  cardinal : unit -> int; }
```

For instance, we use an implementation of polymorphic, persistent sets (implemented as red-black trees), as a replacement for lists.

Not holding sets in memory A big source of inefficiency is the call to the cartesian_product function above (§1). We hold in memory at the same time all possible products, then pipe them into the function calls so as to generate an even bigger set of elements. Only when the set of all elements has been constructed do we actually run destruct, only to extract the instances that we have created in the process. Holding in memory the set of all possible products is too expensive. We adopt instead a symbolic representation of sets, where unions and products are explicitly represented using constructors. This mirrors our algebra of positive types.

```ocaml
type _ set =
  | Set : 'a bag * ('a, 'b) bijection * 'b set
  | Union : 'a bag + 'b bag -> ('a, 'b) sum set
  | Product : 'a bag * 'b bag -> ('a * 'b) set
```

This does not suppress the combinatorial explosion. The instance space is still exponentially large; what we gained by changing our representation is that we no longer hold all the “intermediary” instances in memory simultaneously. This allows us to write an iterate function that constructs the various instances on-the-fly.

```ocaml
let rec iterate : type a. (a -> unit) -> a set set -> unit =
  fun f s -> match s with
  | Set : 'a bag -> 'a set
    | Bij : 'a bag * ('a, 'b) bijection -> 'b set
    | Union : 'a bag + 'b bag -> ('a, 'b) sum set
    | Product : 'a bag * 'b bag -> ('a * 'b) set
```

Let us first turn to the case of values. In order to understand what ArtiCheck ought to do, one may ask themselves what the user can do with values. The user may destruct them: given a pair of type 'a * 'b, the user may keep the first element only, thus obtaining a new 'a. The same goes for sums. We thus provide a destruct function, which breaks down positives types by pattern-matching, populating the descriptions of the various types it encounters as it goes. (The remember function records all instances we haven’t encountered yet in the type descriptor ty.)

Keeping this in mind, we must realize that if a function takes an 'a, the user may use any 'a it can produce to call the function. For instance, in the case that 'a is a product type 'a1 * 'a2, then any pair of 'a1 and 'a2 may work. We introduce a function called produce, which reflects the fact the user may choose any possible pair: the function exhibits the entire set of instances we can build for a given type.
Sampling sets

The above optimizations make it possible to build in a relatively efficient manner sets of instances that can be constructed using a small amount of function calls (let’s call them small instances). That is, we naturally implement a breadth-first search of the instance space, which is unlikely to produce many interesting test-cases before we reach a size limit. Indeed, the distribution of instances is skewed: there are more instances obtained after n calls than there are after n+1 calls. It may thus be the case that by the time we reach three of four consecutive function calls, we’ve hit the maximum limit of instances allowed for the type, since it often is the case that the number of instances grows exponentially.

To solve this issue, we turn to reservoir sampling, which is a method to choose a sample of elements from a set that is typically too big to be held in memory. The solution from the literature is a variant of Knuth’s shuffle, and is quite elegant. The idea is as follows: build a collection of k elements; then for each element e of index i such that k < i, pick a number j at random between 1 and i: if j is less than k, replace the j-th element of the collection by e, and do nothing otherwise. In the end, each element that was added has the same probability of being in the collection.

Unfortunately, iterating over the collection to produce new instances of types biases the generation towards small instances. To understand why, imagine that the collection contains initially many 1 and that we produce new elements for the sampling process by taking the sum of two elements of the collection. The final distribution is very likely to be skewed toward small integers...

Sampling sets, done right

What we are looking for is a set-like data-structure, that can be used to sample the elements that are added to it. This can be implemented quite simply using a hash-set, with fixed size buckets. The idea is that when a bucket becomes full, we drop one element. That way, we manage to keep a good balance between the size of our instances sets, and the diversity of the instances.

We have experimented with the three container structures described above: “regular” sets, “sampled” sets and hash-sets. Out of the three, the latest is the one that gives the most interesting results empirically. However, it is likely that other kind of containers, or other tunings of the exploration procedures could make “interesting” instances pop up early.

### 3.3 Instance generation as a fixed point computation

The [apply/destroy] combination only demonstrates how to generate new instances from one specific element of the signature. We need to iterate this on the whole signature, by feeding the new instances by applying any function of the signature. This doesn’t work as is: since there will be multiple values of ‘a (we generate instances of different types simultaneously), type mismatches are to be expected. One could, after all, use yet another GADT and hide the ‘a type parameter behind an existential variable.

```ocaml
module Fix = sig
  type valuation = variable -> property
  type rhs = valuation -> property
  type equations = variable -> rhs
  val lfp: equations -> valuation end
```

A system of equations maps a variable to a right-hand side. Each right-hand side can be evaluated by providing a valuation so as to obtain a property. Valuations map variables to properties. Solving a system of equations amounts to calling the lfp function which, given a set of equations, returns the corresponding valuation.

A perhaps tempting way to fit in this setting would be to define variables to be our ‘a ty (type descriptor) and properties to be ‘a lists (the instances we have built so far); the equations derived from any signature would then describe ways of obtaining new instances by applying any function of the signature. This doesn’t work as is: since there will be multiple values of ‘a (we generate instances of different types simultaneously), type mismatches are to be expected. One could, after all, use yet another GADT and hide the ‘a type parameter behind an existential variable.

```
type variable = Atom : 'a ty -> variable

let () = assert (None = counter_example 'a positive -> ('a -> bool) -> 'a option)
```

The function takes a description of some (positive) datatype ‘a, iterates on the generated instances of this type and checks that a predicate ‘a -> bool holds for all instances, or returns a counter-example otherwise. At a more abstract level, this means that we are checking a property of the form ∀(z ∈ t), T(z) where T(z) is simply a boolean expression. Multiple quantifiers can be simulated through the use of product types, such as in the typical formula of association maps:

```
∀(m ∈ map(K, V), k ∈ K, v ∈ V), find(k, add(k, v, m)) = v
```

which can be expressed as follows (where *@[0] is the operator for creating product type descriptors):

```
let lookup_insert_prop (k, v, m) = lookup k (insert k v m) = v
```

One then naturally wonders what a good language would be for describing the correctness properties we wish to check. In the example above, we naturally veered towards first-order logic, so as to express formulas with only `prex` universal quantification. The universal quantifiers are to be understood with a “test semantics”, that is, they mean to quantify over all the random instances we generated. Can we do better? In particular, can we capture the

### 4. Expressing correctness properties

We mentioned earlier ([2]) the counter_example function.

```
val counter_example: 'a pos -> ('a -> bool) -> 'a option
```

The function takes a description of some (positive) datatype ‘a, iterates on the generated instances of this type and checks that a predicate ‘a -> bool holds for all instances, or returns a counter-example otherwise. At a more abstract level, this means that we are checking a property of the form ∀(z ∈ t), T(z) where T(z) is simply a boolean expression. Multiple quantifiers can be simulated through the use of product types, such as in the typical formula of association maps:

\[ \forall(m \in \text{map}(K, V), k \in K, v \in V), \text{find}(k, \text{add}(k, v, m)) = v \]

which can be expressed as follows (where *@[0] is the operator for creating product type descriptors):

```
let lookup_insert_prop (k, v, m) = lookup k (insert k v m) = v
```

One then naturally wonders what a good language would be for describing the correctness properties we wish to check. In the example above, we naturally veered towards first-order logic, so as to express formulas with only `prex` universal quantification. The universal quantifiers are to be understood with a “test semantics”, that is, they mean to quantify over all the random instances we generated. Can we do better? In particular, can we capture the
The (abridged) interface exported by red-black trees is as follows.

module type RBT = sig

  type 'a t

  val empty : 'a t

  val insert : 'a -> 'a t -> 'a t

  type direction = Left | Right

  val zip_open : 'a t -> 'a t

  val move : direction -> 'a t -> 'a t

end

This example highlights several strengths of ArtiCheck.

First, two different types are involved: the type of trees and the type of zippers. While an aficionado of internal testing may use the empty and insert functions repeatedly to create new instances of 'a t, it becomes harder to type-check calls to either insert or zip_open. Our framework, thanks to GADTs, generates instances of both types painlessly and automatically.

Second, we argue that a potential mistake is detected trivially by ArtiCheck, while it may turn out to be harder to detect using internal testing. If one removes the comments, the signature reveals that pointers into a tree are made up of a zipper along with a tree itself. It seems fairly natural that the developer would want to reveal the zipper type; it is, after all, a fundamental feature of the module. An undercaffeinated developer, when writing internal test functions, would probably perform sequences of calls to the various functions. What they would fail to do, however, is destructing pairs so as to produce a zipper associated with the wrong tree. This particularly wicked usage would probably be overlooked. ArtiCheck successfully destroys the pair and performs recombinations, thus finding the bug.

5.2 Binary Decision Diagrams

Binary Decision Diagrams (BDDs) represent trees for deciding logical formulas. The defining characteristic of BDDs is that they enforce maximal sharing: wherever two structurally equal sub-formulas appear, they are guaranteed to refer to the same object in memory. A consequence is that performing large numbers of function calls does not necessarily mean using substantially more memory: it may very well be the case that significant sharing occurs.

We mentioned earlier that our strategy for external testing amounted, in essence, to representing series of well-typed function calls in the simply typed lambda calculus using in GADT. If we only did that and skipped section §3, externally-testing BDDs would be infeasible, as we would end up representing a huge number of function calls in memory.

Conversely, with the design we exposed earlier, we merely record new instances as they appear without holding the entire set of potential function calls in memory. This allows for an efficient, non-redundant generation of test cases (instances).

5.3 AVL trees

AVL trees are a classic of programming interviews; many a graduate student has been scared by the mere mention of them. It turns out that tenured professors should be scared too: the OCaml implementation of sets, written using AVL trees by a respectable researcher, contained a bug that went unnoticed for more than ten years. The bug was discovered when another enthusiastic researcher set out to formalize the said library in Coq. The bug was fixed, and all was well. Out of curiosity, we decided to run ArtiCheck on the faulty version of the library. After registering only four functions with ArtiCheck, the bug was correctly identified by our library, with arguably less pain than the full Coq formalization required.

6. Related and Future Work

Genericity of value generation The idea of generating random sequences of operations instead of random internal values is not novel; for example, QuickCheck was used as is to test imperative programs [6], by generating random values of an AST of operations, paired to a monadic interpreter of those syntactic descriptions. However, those examples in the literature only involve operations for a single return type, corresponding to the return type of the AST evaluation function. To integrate operations of distinct return types in the same interface description, one needs GADTs or some other form of type-level reasoning.

When multiple value types are involved, we found it helpful to think of well-typed value generation as term/proof search. Our well-typed rule to generate random values at type τ from a function at type σ → τ and random values at type σ could be expressed, in term of QuickCheck Arbitrary instances, as a deduction rule of the form

instance Arbitrary b, Arbitrary (a -> b) => Arbitrary b

However, Haskell’s type-class mechanism would not allow this instance deduction rule, as it does not respect its restrictions for principled, coherent instance elaboration. Type classes are a helpful and convenient host-language mechanism, but they are designed for code inference rather than arbitrary proof search. Our library-level implementation of well-typed proof search using GADTs gives us more freedom, and is the central idea of ArtiCheck.

It is of course possible to see chaining of function/method calls as a metaprogramming problem, and generate a refit description of those calls, to interpret through an external script or reflection/ILT capability, as done in the testing tool Randoop [10]. Doing the generation as a richly-typed host language library gives us stronger type safety guarantees: even if our value generator is buggy, it will never compose operations in a type-incorrect way.
Testing of higher-order or polymorphic interfaces  The type description language we use captures a first-order subset of the simply-typed lambda-calculus. A natural question is whether it would be possible to support random function generation – embed negative types into positives. A simple way to generate a function of type \( \tau \rightarrow \alpha \) is to just generate a \( \alpha \) at each call; QuickCheck additionally uses the \( \tau \) argument to produce additional entropy. This is not completely satisfying as it does not use the argument at type \( \tau \) (which may not be otherwise reachable from the interface) to produce new values of type \( \alpha \). To have full test coverage for higher-order functional, one should locally add the argument to the set of known elements at type \( \tau \), and re-generate values at type \( \alpha \) in that extended environment.

It would also be interesting to support representation of polymorphic operations; we currently only describe monomorphic instantiations. Bernardy et. al. \[3\] have proposed a parametricity-based technique to derive specific monomorphic instances for type arguments, which also reduces the search space of values to be tested. Supporting this technique would be a great asset of a testing library, but it is definitely not obvious how their pen-and-paper derivation could be automatized, especially as a library function.

Bottom-up or top-down generation We have presented the ArtiCheck implementation as a bottom-up process: from a set of already-discovered values at the types of interest, we use the constructors of the interface to produce new values. In contrast, most random checking tools present generation in a top-down fashion: pick the head constructor of the data value, then generate its subcomponents recursively. One notable exception is SmallCheck \[12\], which performs exhaustive testing for values of bounded depth.

The distinction is however blurred by several factors. Fix implements demand-driven computation of fixpoints: if you request elements at type \( \alpha \) and there is an operation \( \tau \rightarrow \alpha \), it will recursively populate values at type \( \tau \), giving the actual operational behavior of the generator a top-down flavor. Relatedly, SmallCheck has a Lazy SmallCheck variant that uses laziness (demand-driven computation) to avoid fleshing out value parts that are not inspected by the property being tested.

Furthermore, the genericity of our high-level interface makes ArtiCheck amenable to a change in the generation technique; we could implement direct top-down search without changing the signature description language, or most parts of the library interface.

Richer property languages We discussed in Section 3 the difficulty of isolating an expressive fragment of first-order logic as a property language that could be given a realizable testing semantics. As it performs exhaustive search (up to a bound), SmallCheck is able to give a non-surprising semantics to existential quantification. As we set user control for each interface datatype whether an exhaustive collection or a sampling collection should be used, we could support existential on exhaustively collected types only.

In related manner, Berghofer and Nipkow’s implementation of QuickCheck for Isabelle \[2\] stands out by supporting full-fledged first-order logic for random checking. In the Isabelle proof assistant, it is common to define computations as inductive relations/predicates that can be given a (potentially non-deterministic) functional mode; instead of directly turning correctness formulas into testing programs, they translate formulas into inductive datatypes, which are then given a computational interpretation.

This is remarkable as it not only allows them to support a rich specification language, but also gives a principled explanation for the ad-hoc semantics of preconditions in testing frameworks (a failing precondition does not count as a passed test); instead of seeing a precondition \( P[x] \) as returning a boolean from a randomly-generated \( x \), they choose a mode assignment that inverts it into a logic program generating the \( x \)s accepted by the precondition. This gives a logic-based justification to various heuristics used in other works to generate random values more likely to pass the precondition, either domain-specific \[7\] or SAT-based \[1\].

Conclusion We have presented the design of ArtiCheck, a novel library that allows one to check the invariants of a module signature by simulating user interaction with the module. ArtiCheck behaves like a fake client: it calls functions, constructs and destructs products or sums, and for each element check that the invariants are verified. The key to performing this in a generic, abstract manner relies on GADTs, which abstract the different types that may be manipulated into a common representation.

We identified various performance problems that arise. The library handles them via a symbolic representation of types in combination with a little bit of mutable state to avoid handling large, intermediary results in memory.

The result is a self-contained library that wraps the core concepts of external testing and offers clients a cheap and efficient way to test their programs. The library, for instance, successfully detects infamous issues such as the AVL re-balancing issue in the standard library of OCaml, with a much lower cost than a complete machine- assisted verification of the module.

While the library exposes the essence of external testing and has already proven worthwhile, we believe there is potential for improvement and expansion into a fully-fledged testing library. The code, along with the entire history of the present paper, is available online at https://github.com/braibant/articheck

References