MPRI, Typage

Didier Rémy
(With course material from François Pottier)

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Plan of the course

Introduction

Simply-typed λ-calculus

Polymorphism and System F

Type reconstruction

Existential types

Existential types

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Type-preserving compilation

Compilation is type-preserving when each intermediate language is *explicitly typed*, and each compilation phase transforms a typed program into a typed program in the next intermediate language.

Why *preserve types* during compilation?

- it can help debug the compiler;
- types can be used to drive optimizations;
- types can be used to produce *proof-carrying code;*
- proving that types are preserved can be the first step towards proving that the *semantics* is preserved [Chlipala, 2007].

Type-preserving compilation

Type-preserving compilation exhibits an encoding of programming constructs into programming languages with usually richer type systems.

The encoding may sometimes be used directly as a programming idiom in the source language.

For example:

- Closure conversion requires an extension of the language with existential types, which happens to be very useful on their own.
- Closures are themselves a simple form of objects.
- Defunctionalization may be done manually on some particular programs, *e.g.* in web applications to monitor the computation.

Type-preserving compilation

A classic paper by Morrisett *et al.* [1999] shows how to go from System F to Typed Assembly Language, while preserving types along the way. Its main passes are:

- *CPS conversion* fixes the order of evaluation, names intermediate computations, and makes all function calls tail calls;
- *closure conversion* makes environments and closures explicit, and produces a program where all functions are closed;
- allocation and initialization of tuples is made explicit;
- the calling convention is made explicit, and variables are replaced with (an unbounded number of) machine registers.

Translating types

In general, a type-preserving compilation phase involves not only a translation of *terms,* mapping M to $[\![M]\!]$, but also a translation of *types,* mapping τ to $[\![\tau]\!]$, with the property:

$$\Gamma \vdash M : \tau \quad \text{implies} \quad [\![\Gamma]\!] \vdash [\![M]\!] : [\![\tau]\!]$$

The translation of types carries a lot of information: examining it is often enough to guess what the translation of terms will be.

Contents

Closure conversion

First-class functions may appear in the body of other functions. hence, their own body may contain free variables that will be bound to values during the evaluation in the execution environment.

Because they can be returned as values, and thus used outside of their definition environment, they must store their execution environment in their value.

A *closure* is the packaging of the code of a first-class function with its runtime environment, so that it becomes closed, *i.e.* independent of the runtime environment and can be moved and applied in another runtime environment.

Closures can also be used to represent recursive functions and objects (in the object-as-record-of-methods paradigm).

Source and target

In the following,

- the *source* calculus has *unary* λ-abstractions, which can have free variables;
- the *target* calculus has *binary* λ-abstractions, which must be *closed.*

Closure conversion can be easily extended to n-ary functions, or n-ary functions may be *uncurried* in a separate, type-preserving compilation pass.

Variants of closure conversion

There are at least two variants of closure conversion:

- in the *closure-passing variant*,
  the closure and the environment are a single memory block;
- in the *environment-passing variant*,
  the environment is a separate block, to which the closure points.

The impact of this choice on the translation of terms is minor.

Its impact on the translation of types is more important:
the closure-passing variant requires more type-theoretic machinery.

Closure-passing closure conversion

Let $\{x_1, \ldots, x_n\}$ be $\mathrm{fv}(\lambda x.\, a)$:

$$
\begin{array}{rcl}
[\![\lambda x.\, a]\!] & = & \mathit{let}\ \mathit{code} = \lambda(\mathit{clo}, x).\ \\
& & \quad \mathit{let}\ (\_, x_1, \ldots, x_n) = \mathit{clo}\ \mathit{in}\ [\![a]\!]\ \mathit{in}\ \\
& & (\mathit{code}, x_1, \ldots, x_n) \\
[\![a_1\ a_2]\!] & = & \mathit{let}\ \mathit{clo} = [\![a_1]\!]\ \mathit{in}\ \\
& & \mathit{let}\ \mathit{code} = \mathit{proj}_0\ \mathit{clo}\ \mathit{in}\ \\
& & \mathit{code}\ (\mathit{clo}, [\![a_2]\!])
\end{array}
$$

(The variables code and clo must be suitably fresh.)

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Important! The layout of the environment must be known only at the closure allocation site, not at the call site. In particular, $\mathit{proj}_0\ \mathit{clo}$ need not know the size of clo.

Environment-passing closure conversion

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[\![\lambda x.\, a]\!] & = & \mathit{let}\ code = \lambda(env, x). \\
& & \quad \mathit{let}\ \ (x_1, \ldots, x_n) = env\ \mathit{in}\ [\![a]\!]\ \mathit{in} \\
& & (code, (x_1, \ldots, x_n)) \\
[\![a_1\ a_2]\!] & = & \mathit{let}\ (code, env) = [\![a_1]\!]\ \mathit{in} \\
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Questions: How can closure conversion be made *type-preserving?*

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Questions: How can closure conversion be made *type-preserving?*

The key issue is to find a sensible definition of the type translation. In particular, what is the translation of a function type, $[\![\tau_1 \to \tau_2]\!]$?

Environment-passing closure conversion

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Assume $\Gamma \vdash \lambda x.\, a : \tau_1 \to \tau_2$.
Assume, *w.l.o.g.*. $\mathrm{dom}(\Gamma) = \mathrm{fv}(\lambda x.\, a) = \{x_1, \ldots, x_n\}$.

Write $[\![\Gamma]\!]$ for the tuple type $x_1 : [\![\tau_1']\!]; \ldots; x_n : [\![\tau_n']\!]$ where Γ is $x_1 : \tau_1'; \ldots; x_n : \tau_n'$. We also use $[\![\Gamma]\!]$ as a type to mean $[\![\tau_1']\!] \times \ldots \times [\![\tau_n']\!]$.

We have $\Gamma, x : \tau_1 \vdash a : \tau_2$, so in environment $[\![\Gamma]\!], x : [\![\tau_1]\!]$, we have

- env has type $[\![\Gamma]\!]$,
- code has type $([\![\Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!]$, and
- the entire closure has type $(([\![\Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times [\![\Gamma]\!]$.

Now, *what should be the definition of* $[\![\tau_1 \to \tau_2]\!]$*?*

Towards a type translation

Can we adopt this as a definition?

$$[\![\tau_1 \to \tau_2]\!] \quad = \quad (([\![\Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times [\![\Gamma]\!]$$

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Naturally not. This definition is mathematically ill-formed: we cannot use Γ out of the blue.

Hmm... Do we really need to have a uniform translation of types?

Towards a type translation

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We need a uniform translation of types, not just because it is nice to have one, but because it describes a *uniform calling convention.*

If closures with distinct environment sizes or layouts receive distinct types, then we will be unable to translate this well-typed code:

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We need a uniform translation of types, not just because it is nice to have one, but because it describes a *uniform calling convention.*

If closures with distinct environment sizes or layouts receive distinct types, then we will be unable to translate this well-typed code:

$$\textit{if } \ldots \textit{ then } \lambda x.\, x + y \textit{ else } \lambda x.\, x$$

Furthermore, we want function invocations to be translated uniformly, without knowledge of the size and layout of the closure's environment.

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Furthermore, we want function invocations to be translated uniformly, without knowledge of the size and layout of the closure's environment.

So, *what could be the definition of* $[\![\tau_1 \to \tau_2]\!]$*?*

The type translation

The only sensible solution is:

$$[\![\tau_1 \to \tau_2]\!] \quad = \quad \exists\alpha.((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times \alpha$$

An *existential quantification* over the type of the environment abstracts away the differences in size and layout.

Enough information is retained to ensure that the application of the code to the environment is valid: this is expressed by letting the variable α occur twice on the right-hand side.

The type translation

The existential quantification also provides a form of *security*: the caller cannot do anything with the environment except pass it as an argument to the code; in particular, it cannot inspect or modify the environment.

For instance, in the source language, the following coding style guarantees that x remains even, no matter how f is used:

$$\mathit{let}\ f = \mathit{let}\ x = \mathit{ref}\ 0\ \mathit{in}\ \lambda().\, x := (x + 2); !\, x$$

After closure conversion, the reference x is reachable via the closure of f.
A malicious, untyped client could write an odd value to x.
However, a *well-typed* client is unable to do so.

This encoding is not just type-preserving, but also *fully abstract:* it preserves (a typed version of) observational equivalence [Ahmed and Blume, 2008].

Contents

Existential types

One can extend System F with *existential types,* in addition to universals:

$$\tau ::= \ldots \mid \exists \alpha.\tau$$

As in the case of universals, there are *type-passing* and *type-erasing* interpretations of the terms and typing rules... and in the latter interpretation, there are *explicit* and *implicit* versions.

Let's just look at the type-erasing interpretation, with an explicit notation for introducing and eliminating existential types.

Existential types in explicit style

Here is how the existential quantifier is introduced and eliminated:

$$\textsc{Pack}\quad \frac{\Gamma \vdash M : [\alpha \mapsto \tau']\tau}{\Gamma \vdash \mathit{pack}\,\tau', M\ \mathit{as}\ \exists\alpha.\tau : \exists\alpha.\tau}$$

$$\textsc{Unpack}\quad \frac{\Gamma \vdash M_1 : \exists\alpha.\tau_1 \qquad \Gamma, \alpha, x : \tau_1 \vdash M_2 : \tau_2}{\Gamma \vdash \mathit{let}\,\alpha, x = \mathit{unpack}\,M_1\ \mathit{in}\ M_2 : \tau_2}$$

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Anything wrong?

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The side condition $\alpha \# \tau_2$ is mandatory here to ensure well-formedness of the conclusion.

The side condition may also be written $\Gamma \vdash \tau_2$ which implies $\alpha \# \tau_2$, given that the well-formedness of the last premise implies $\alpha \notin \mathrm{dom}(\Gamma)$.

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Note the *imperfect* duality between universals and existentials:

$$\textsc{TAbs}\ \frac{\Gamma, \alpha \vdash M : \tau}{\Gamma \vdash \Lambda\alpha.M : \forall\alpha.\,\tau} \qquad \textsc{TApp}\ \frac{\Gamma \vdash M : \forall\alpha.\,\tau}{\Gamma \vdash M\,\tau' : [\alpha \mapsto \tau']\tau}$$

On existential elimination

It would be nice to have a simpler elimination form, perhaps like this:

$$\frac{\Gamma, \alpha \vdash M : \exists \alpha.\tau}{\Gamma, \alpha \vdash \textit{unpack}\, M : \tau}$$

Informally, this could mean that, if M has type τ for some *unknown* α, then it has type τ, where α is "fresh"...

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Why is this broken?

We can immediately *universally* quantify over α, and conclude that $\Gamma \vdash \Lambda\alpha.\textit{unpack}\, M : \forall \alpha.\, \tau$. This is nonsense!

Replacing the premise $\Gamma, \alpha \vdash M : \exists \alpha.\tau$ by the conjunction $\Gamma \vdash M : \exists \alpha.\tau$ and $\alpha \in \mathrm{dom}(\Gamma)$ would make the rule even more permissive, so it wouldn't help.

On existential elimination

A correct elimination rule must force the existential package to be *used* in a way that does not rely on the value of α.

Hence, the elimination rule must have control over the *user* of the package – that is, over the term M_2.

$$\textsc{Unpack}\quad \frac{\Gamma \vdash M_1 : \exists \alpha.\tau_1 \qquad \Gamma, \alpha; x : \tau_1 \vdash M_2 : \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash \mathit{let}\ \alpha, x = \mathit{unpack}\ M_1\ \mathit{in}\ M_2 : \tau_2}$$

The restriction $\alpha \# \tau_2$ prevents writing "*let* $\alpha, x =$ *unpack* M_1 *in* x", which would be equivalent to the unsound "*unpack* M" of previous slide.

The fact that α is bound within M_2 forces it to be treated abstractly.

In fact, M_2 must be ??? in α.

On existential elimination

In fact, M_2 must be *polymorphic* in α: the rule could be written

$$\frac{\Gamma \vdash M_1 : \exists\alpha.\tau_1 \qquad \Gamma \vdash \Lambda\alpha.\lambda x.\, M_2 : \forall\alpha.\, \tau_1 \to \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash \mathit{let}\ \alpha, x = \mathit{unpack}\ M_1\ \mathit{in}\ M_2 : \tau_2}$$

or, if N_2 is $\Lambda\alpha.\lambda x.\, M_2$:

$$\frac{\Gamma \vdash M_1 : \exists\alpha.\tau_1 \qquad \Gamma \vdash N_2 : \forall\alpha.\, \tau_1 \to \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash \mathit{unpack}\ M_1\ N_2 : \tau_2}$$

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One could even view "$\mathit{unpack}_{\exists\alpha.\tau_1}$" as a *constant* with all these types:

$$\mathit{unpack}_{\exists\alpha.\tau_1} : \quad (\exists\alpha.\tau_1) \to \big(\forall\alpha.\,(\tau_1 \to \tau_2)\big) \to \tau_2 \qquad \alpha \# \tau_2$$

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Thus, $\mathit{unpack}_{\exists\alpha.\tau} : \quad \forall\beta.\big((\exists\alpha.\tau) \to (\forall\alpha.\,(\tau \to \beta)) \to \beta\big)$

or, better $\mathit{unpack}_{\exists\alpha.\tau} : \quad (\exists\alpha.\tau) \to \forall\beta.\big((\forall\alpha.\,(\tau \to \beta)) \to \beta\big)$

β stands for τ_2: it is bound prior to α, so it cannot be instantiated to a type that refers to α, which reflects the side condition $\alpha \# \tau_2$.

On existential introduction

$$\textsc{Pack}\ \frac{\Gamma \vdash M : [\alpha \mapsto \tau']\tau}{\Gamma \vdash \mathit{pack}\,\tau', M\ \mathit{as}\ \exists\alpha.\,\tau : \exists\alpha.\tau}$$

If desired, "$\mathit{pack}_{\exists\alpha.\tau}$" could also be viewed as a *constant* with all the types:

$$\mathit{pack}_{\exists\alpha.\tau} : \quad [\alpha \mapsto \tau']\tau \to \exists\alpha.\tau$$

i.e. with polymorphic type:

$$\mathit{pack}_{\exists\alpha.\tau} : \quad \forall\alpha.\,(\tau \to \exists\alpha.\tau)$$

Existentials as constants

In System F, existential types can also be presented as constants

$$\begin{aligned} \mathit{pack}_{\exists\alpha.\tau} &: \forall\alpha.\,(\tau \to \exists\alpha.\tau) \\ \mathit{unpack}_{\exists\alpha.\tau} &: \exists\alpha.\,\tau \to \forall\beta.\,((\forall\alpha.\,(\tau \to \beta)) \to \beta) \end{aligned}$$

Read:

- for *any* α, if you have a τ, then, for *some* α, you have a τ;
- if, for *some* α, you have a τ, then, (for any β,) if you wish to obtain a β out of it, then you must present a function which, for *any* α, obtains a β out of a τ.

This is somewhat reminiscent of ordinary first-order logic:
$\exists x.F$ is equivalent to, and can be defined as, $\neg(\forall x.\,\neg F)$.

Is there an encoding of existential types into universal types?

Encoding existentials into universals

The type translation is *double negation:*

$$[\![\exists\alpha.\tau]\!] \;=\; \forall\beta.((\forall\alpha.([\![\tau]\!]\to\beta))\to\beta) \qquad \text{if } \beta\#\tau$$

The term translation is:

$$\begin{array}{rcl} [\![\mathit{pack}_{\exists\alpha.\tau}]\!] & : & \forall\alpha.([\![\tau]\!]\to[\![\exists\alpha.\tau]\!]) \\ & = & ? \end{array}$$

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$$[\![\exists\alpha.\tau]\!] \;=\; \forall\beta.((\forall\alpha.([\![\tau]\!]\to\beta))\to\beta) \qquad \text{if } \beta\#\tau$$

The term translation is:

$$\begin{array}{rcl} [\![\mathit{pack}_{\exists\alpha.\tau}]\!] & : & \forall\alpha.([\![\tau]\!]\to[\![\exists\alpha.\tau]\!]) \\ & = & \Lambda\alpha.\lambda x{:}[\![\tau]\!].\,\Lambda\beta.\lambda k{:}\forall\alpha.([\![\tau]\!]\to\beta).\,k\;\alpha\;x \\ [\![\mathit{unpack}_{\exists\alpha.\tau}]\!] & : & [\![\exists\alpha.\tau]\!]\to\forall\beta.((\forall\alpha.([\![\tau]\!]\to\beta))\to\beta) \\ & = & \lambda x{:}[\![\exists\alpha.\tau]\!].\,x \end{array}$$

There is little choice, if the translation is to be type-preserving.

What is the computational content of this encoding?

Encoding existentials into universals

The type translation is *double negation:*

$$[\![\exists\alpha.\tau]\!] \;=\; \forall\beta.((\forall\alpha.([\![\tau]\!]\to\beta))\to\beta) \qquad \text{if } \beta\#\tau$$

The term translation is:

$$\begin{array}{rcl} [\![\mathit{pack}_{\exists\alpha.\tau}]\!] & : & \forall\alpha.([\![\tau]\!]\to[\![\exists\alpha.\tau]\!]) \\ & = & \Lambda\alpha.\lambda x{:}[\![\tau]\!].\,\Lambda\beta.\lambda k{:}\forall\alpha.([\![\tau]\!]\to\beta).\,k\;\alpha\;x \\ [\![\mathit{unpack}_{\exists\alpha.\tau}]\!] & : & [\![\exists\alpha.\tau]\!]\to\forall\beta.((\forall\alpha.([\![\tau]\!]\to\beta))\to\beta) \\ & = & \lambda x{:}[\![\exists\alpha.\tau]\!].\,x \end{array}$$

There is little choice, if the translation is to be type-preserving.

What is the computational content of this encoding?

A *continuation-passing transform.*

This encoding is due to Reynolds [1983],
although it has more ancient roots in logic.

The semantics of existential types as constants

$\mathit{pack}_{\exists\alpha.\tau}$ can be treated as a unary constructor, and $\mathit{unpack}_{\exists\alpha.\tau}$ as a unary destructor. The δ-reduction rule is:

$$\mathit{unpack}_{\exists\alpha.\tau_0}\ (\mathit{pack}_{\exists\alpha.\tau}\tau'\ V)\ \longrightarrow\ \Lambda\beta.\lambda y{:}\forall\alpha.\tau\to\beta.\, y\ \tau'\ V$$

It would be more intuitive, however, to treat $\mathit{unpack}_{\exists\alpha.\tau_0}$ as a binary destructor:

$$\mathit{unpack}_{\exists\alpha.\tau_0}\ (\mathit{pack}_{\exists\alpha.\tau}\tau'\ V)\ \tau_1\ (\Lambda\alpha.\lambda x{:}\tau.\, M)\ \longrightarrow\ [\alpha\mapsto\tau'][x\mapsto V]M$$

This does not quite fit in our generic framework for constants, which must receive all type arguments prior to value arguments.

But our framework could be extended.

The semantics of existential types as primitive

We extend values and evaluation contexts as follows:

$$\begin{array}{rcl} V & ::= & \ldots \mathit{pack}\,\tau', V\ \mathit{as}\,\tau \\ E & ::= & \ldots \mathit{pack}\,\tau', [\,]\ \mathit{as}\,\tau \mid \mathit{let}\,\alpha, x = \mathit{unpack}\,[\,]\ \mathit{in}\ M \end{array}$$

We add the reduction rule:

$$\mathit{let}\,\alpha, x = \mathit{unpack}\,(\mathit{pack}\,\tau', V\ \mathit{as}\,\tau)\ \mathit{in}\ M \longrightarrow [\alpha \mapsto \tau'][x \mapsto V]M$$

Exercise

Show that subject reduction and progress hold.

The semantics of existential types beware!

The reduction rule for existentials destructs its arguments.

Hence, *let* $\alpha, x =$ *unpack* M_1 *in* M_2 cannot be reduced unless M_1 is itself a packed expression, which is indeed the case when M_1 is a value (or in head normal form).

This contrasts with *let* $x : \tau = M_1$ *in* M_2 where M_1 need not be evaluated and may be an application (*e.g.* with call-by-name or strong reduction strategies).

The semantics of existential types beware!

Exercise

Find an example that illustrates why the reduction of
let $\alpha, x =$ *unpack* M_1 *in* M_2 *could be problematic when* M_1 *is not a value.*

The semantics of existential types beware!

Exercise

Find an example that illustrates why the reduction of
let $\alpha, x =$ *unpack* M_1 *in* M_2 *could be problematic when* M_1 *is not a value.*

Need a hint?

Use a conditional

The semantics of existential types beware!

Exercise

Find an example that illustrates why the reduction of
let $\alpha, x =$ *unpack* M_1 *in* M_2 *could be problematic when* M_1 *is not a value.*

Solution

Let M_1 be if M then V_1 else V_2 where V_i is of the form
pack τ_i, V_i *as* $\exists\alpha.\tau$ and the two witnesses τ_1 and τ_2 differ.

There is no common type for the unpacking of the two possible results V_1 and V_2. The choice between those two possible results must be made, by evaluating M_1, before unpacking.

Is pack too verbose?

Exercise

Recall the typing rule for pack:

$$\frac{\Gamma \vdash M : [\alpha \mapsto \tau']\tau}{\Gamma \vdash \mathit{pack}\, \tau', M \ \mathit{as}\ \exists\alpha.\, \tau : \exists\alpha.\tau}$$

Isn't the witness type τ' annotation superfluous?

Is pack too verbose?

Exercise

Recall the typing rule for pack:

$$\frac{\Gamma \vdash M : [\alpha \mapsto \tau']\tau}{\Gamma \vdash \mathit{pack}\,\tau', M\ \mathit{as}\ \exists\alpha.\,\tau : \exists\alpha.\tau}$$

Isn't the witness type τ' annotation superfluous?

- The type τ_0 of M is fully determined by M. Given the type $\exists\alpha.\tau$ of the packed value, checking that τ_0 is of the form $[\alpha \mapsto \tau']\tau$ is the matching problem for second-order types, which is simple.
- However, the reduction rule need the witness type τ'. If it were not available, it would have to be computed during reduction. The reduction rule would then not be pure rewriting.

The explicitly-typed language need the witness type for simplicity, while in the surface language, it could be omitted and reconstructed.

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Implicitly-typed existential types

Intuitively, pack and unpack are just type annotations that could be dropped, leaving a let-binding instead of the unpack form.

Hence, the typing rule for implicitly-typed existential types:

$$\textsc{Unpack}\quad \frac{\Gamma \vdash a_1 : \exists \alpha.\tau_1 \qquad \Gamma, \alpha, x : \tau_1 \vdash a_2 : \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash \mathit{let}\ x = a_1\ \mathit{in}\ a_2 : \tau_2} \qquad \textsc{Pack}\quad \frac{\Gamma \vdash a : [\alpha \mapsto \tau']\tau}{\Gamma \vdash a : \exists \alpha.\tau}$$

Notice, however, that this let-binding is not typechecked as syntactic sugar for an immediate application!

The semantics of this let-binding is as before:

$$E ::= \ldots \mid \mathit{let}\ x = E\ \mathit{in}\ M \qquad\qquad \mathit{let}\ x = V\ \mathit{in}\ M \longrightarrow [x \mapsto V]M$$

Is the semantics type-erasing?

Implicitly-typed existential types

subtlety

Yes, it is.

But there is a subtlety!

Implicitly-typed existential types

subtlety

Yes, it is.

But there is a subtlety! What about the call-by-name semantics?

Implicitly-typed existential types

subtlety

Yes, it is.

But there is a subtlety! What about the call-by-name semantics?

We chose a call-by-value semantics, but so far, as long as there is no side-effect, we could have chosen a call-by-name semantics (or even perform reduction under abstraction).

In a call-by-name semantics, the let-bound expression is not reduced prior to substitution in the body:

$$\mathit{let}\, x = M_1\ \mathit{in}\ M_2 \longrightarrow [x \mapsto M_1]M_2$$

With existential types, this breaks subject reduction!

Why?

Implicitly-typed existential types

subtlety

Let τ_0 be $\exists\alpha.\,(\alpha \to \alpha) \to (\alpha \to \alpha)$ and v_0 a value of type *bool*. Let v_1 and v_2 be two values of type τ_0 with incompatible witness types, *e.g.* $\lambda f.\,\lambda x.\,1 + (f\ (1 + x))$ and $\lambda f.\,\lambda x.\,not\ (f\ (not\ x))$.

Let v be the function $\lambda b.$ if b then v_1 else v_2 of type $\mathit{bool} \to \tau_0$.

$$a_1 \quad = \quad \mathit{let}\ x = v\ v_0\ \mathit{in}\ x\ (x\ (\lambda y.\,y)) \quad \longrightarrow \quad v\ v_0\ (v\ v_0\ (\lambda y.\,y)) \quad = \quad a_2$$

We have $\varnothing \vdash a_1 : \exists\alpha.\,\alpha \to \alpha$ while $\varnothing \not\vdash a_2 : \tau$.

What happened?

Implicitly-typed existential types — subtlety

Let τ_0 be $\exists\alpha.\,(\alpha \to \alpha) \to (\alpha \to \alpha)$ and v_0 a value of type *bool*. Let v_1 and v_2 be two values of type τ_0 with incompatible witness types, *e.g.* $\lambda f.\,\lambda x.\,1 + (f\ (1 + x))$ and $\lambda f.\,\lambda x.\,not\ (f\ (not\ x))$.

Let v be the function $\lambda b.$ if b then v_1 else v_2 of type $\mathit{bool} \to \tau_0$.

$$a_1 \;=\; \mathit{let}\ x = v\ v_0\ \mathit{in}\ x\ (x\ (\lambda y.\,y)) \;\longrightarrow\; v\ v_0\ (v\ v_0\ (\lambda y.\,y)) \;=\; a_2$$

We have $\varnothing \vdash a_1 : \exists\alpha.\,\alpha \to \alpha$ while $\varnothing \nvdash a_2 : \tau$.

The term a_1 is well-typed since $v\ v_0$ has type τ_0, hence x can be assumed of type $(\beta \to \beta) \to (\beta \to \beta)$ for some unknown type β and $\lambda y.\,y$ is of type $\beta \to \beta$.

However, without the outer existential type $v\ v_0$ can only be typed with $(\forall\alpha.\,\alpha \to \alpha) \to \exists\alpha.\,(\alpha \to \alpha)$, because the value returned by the function need different witnesses for α. This is demanding too much on its argument and the outer application is ill-typed.

Implicitly-typed existential types

subtlety

One could wonder whether the syntax should not allow the implicit introduction of unpacking (instead of requesting a let-binding).

One could argue that if some expression is the expansion of a well-typed let-binding, then it should also be well-typed:

$$\frac{\Gamma \vdash a_1 : \exists \alpha.\tau_1 \qquad \Gamma, \alpha, x : \tau_1 \vdash a_2 : \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash [x \mapsto a_1]a_2 : \tau_2}$$

Comments?

Implicitly-typed existential types — subtlety

One could wonder whether the syntax should not allow the implicit introduction of unpacking (instead of requesting a let-binding).

One could argue that if some expression is the expansion of a well-typed let-binding, then it should also be well-typed:

$$\frac{\Gamma \vdash a_1 : \exists \alpha.\tau_1 \qquad \Gamma, \alpha, x : \tau_1 \vdash a_2 : \tau_2 \qquad \alpha \# \tau_2}{\Gamma \vdash [x \mapsto a_1]a_2 : \tau_2}$$

Comments:

- This rule does not have a logical flavor...
- It fixes the previous example, but not the general case:
  Pick a_1 that is not yet a value after one reduction step.
  Then, after let-expansion reduce, one of the two occurrences of a_1.
  The result is no longer of the form $[x \mapsto a_1]a_2$.

Implicitly-typed existential types

subtlety

Existential types are trickier than they may appear at first.

The subject reduction property breaks if reduction is not restricted to expressions in head-normal forms.

Unrestricted reduction is still safe because well-typedness may eventually be recovered by further reduction steps—so that progress will never break.

Implicitly-typed existential types — encoding

Notice that the CPS encoding of existential types (1) enforces the evaluation of the packed value (2) before it can be unpacked (3) and substituted(4):

$$\begin{array}{rcll} [\![\mathit{unpack}\, a_1\ (\lambda x.\, a_2)]\!] & = & [\![a_1]\!]\ (\lambda x.\, [\![a_2]\!]) & \mathbf{(1)} \\ & \longrightarrow & (\lambda k.\, [\![a]\!]\ k)\ (\lambda x.\, [\![a_2]\!]) & \mathbf{(2)} \\ & \longrightarrow & (\lambda x.\, [\![a_2]\!])\ [\![a]\!] & \mathbf{(3)} \\ & \longrightarrow & [x \mapsto [\![a]\!]][\![a_2]\!] & \mathbf{(4)} \end{array}$$

In the call-by-value setting, $\lambda k.\, [\![a]\!]\ k$ would come from the reduction of $[\![\mathit{pack}\, a]\!]$, *i.e.* is $(\lambda k.\, \lambda x.\, k\ x)\ [\![a]\!]$, so that a is always a value v.

However, a need not be a value. What is essential is that a_1 be reduced to some head normal form $\lambda k.\, [\![a]\!]\ k$.

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Iso-existential types in ML

What if one wished to extend ML with existential types?

Full type inference for existential types is undecidable, just like type inference for universals.

However, introducing existential types in ML is easy if one is willing to rely on user-supplied *annotations* that indicate where to pack and unpack.

Iso-existential types in ML

This *iso-existential* approach was suggested by Läufer and Odersky [1994].

Iso-existential types are explicitly *declared:*

$$D\ \vec{\alpha} \approx \exists \bar{\beta}.\tau \qquad \text{if } \mathrm{ftv}(\tau) \subseteq \bar{\alpha} \cup \bar{\beta} \quad \text{and} \quad \bar{\alpha} \# \bar{\beta}$$

This introduces two constants, with the following type schemes:

$$\begin{array}{rcl} \mathit{pack}_D & : & \forall \bar{\alpha}\bar{\beta}.\,\tau \to D\ \vec{\alpha} \\ \mathit{unpack}_D & : & \forall \bar{\alpha}\gamma.\,D\ \vec{\alpha} \to (\forall \bar{\beta}.\,(\tau \to \gamma)) \to \gamma \end{array}$$

(Compare with basic iso-recursive types, where $\bar{\beta} = \varnothing$.)

Iso-existential types in ML

A few corners have been cut on the previous slide. The "type scheme:"

$$\forall \bar{\alpha}\gamma.\, D\ \vec{\alpha} \to (\forall \bar{\beta}.\, (\tau \to \gamma)) \to \gamma$$

is in fact *not* an ML type scheme. How could we address this?

Iso-existential types in ML

A few corners have been cut on the previous slide. The "type scheme:"

$$\forall \bar{\alpha}\gamma.\, D\ \vec{\alpha} \to (\forall \bar{\beta}.\, (\tau \to \gamma)) \to \gamma$$

is in fact *not* an ML type scheme. How could we address this?

A solution is to make unpack_D a binary construct again (rather than a constant), with an *ad hoc* typing rule:

$\textsc{Unpack}_D$

$$\frac{\Gamma \vdash M_1 : D\ \vec{\tau} \qquad \Gamma \vdash M_2 : \forall \bar{\beta}.\, ([\vec{\alpha} \mapsto \vec{\tau}]\tau \to \tau_2) \qquad \bar{\beta} \# \vec{\tau}, \tau_2}{\Gamma \vdash \mathit{unpack}_D\ M_1\ M_2 : \tau_2}$$

where $D\ \vec{\alpha} \approx \exists \bar{\beta}.\tau$

We have seen a version of this rule in System F earlier; this in an ML version. The term M_2 must be polymorphic, which $\textsc{Gen}$ can prove.

Iso-existential types in ML

Iso-existential types are perfectly compatible with ML type inference.

The constant pack_D admits an ML type scheme, so it is unproblematic.

The construct unpack_D leads to this constraint generation rule (see type inference):

$$\langle\!\langle \mathit{unpack}_D\ M_1\ M_2 : \tau_2 \rangle\!\rangle \quad = \quad \exists \bar{\alpha}. \left(\begin{array}{l} \langle\!\langle M_1 : D\ \vec{\alpha} \rangle\!\rangle \\ \forall \bar{\beta}. \langle\!\langle M_2 : \tau \to \tau_2 \rangle\!\rangle \end{array} \right)$$

where $D\ \vec{\alpha} \approx \exists \bar{\beta}.\tau$ and, *w.l.o.g.*, $\bar{\alpha}\bar{\beta} \# M_1, M_2, \tau_2$.

A universally quantified constraint appears where polymorphism is *required.*

Iso-existential types in ML

In practice, Läufer and Odersky suggest fusing iso-existential types with algebraic data types.

This can be done in OCaml using GADTs (see last part of the course). The (somewhat bizarre) syntax for this in OCaml is:

$$\mathit{type}\ D\ \vec{\alpha} = \ell : \tau\ \to D\ \vec{\alpha}$$

where ℓ is a data constructor and $\bar{\beta}$ appears free in τ but does not appear in $\vec{\alpha}$. The elimination construct becomes:

$$\langle\!\langle\, \mathit{match}\ M_1\ \mathit{with}\ \ell\ x \to M_2 : \tau_2 \,\rangle\!\rangle \quad = \quad \exists\bar{\alpha}.\left(\begin{array}{l} \langle\!\langle M_1 : D\ \vec{\alpha} \rangle\!\rangle \\ \forall\bar{\beta}.\, \mathit{def}\ x : \tau\ \mathit{in}\ \langle\!\langle M_2 : \tau_2 \rangle\!\rangle \end{array}\right)$$

where, w.l.o.g., $\bar{\alpha}\bar{\beta} \# M_1, M_2, \tau_2$.

An example

Define $\mathit{Any} \approx \exists\beta.\beta$. An attempt to extract the raw content of a package fails:

$$\begin{array}{rcl} \langle\!\langle \mathit{unpack}_{\mathit{Any}}\, M_1\ (\lambda x.\, x) : \tau_2 \rangle\!\rangle & = & \langle\!\langle M_1 : \mathit{Any} \rangle\!\rangle \wedge \forall\beta.\, \langle\!\langle \lambda x.\, x : \beta \to \tau_2 \rangle\!\rangle \\ & \Vdash & \forall\beta.\, \beta = \tau_2 \\ & \equiv & \mathit{false} \end{array}$$

(Recall that $\beta \# \tau_2$.)

An example

Define

$$D\ \alpha \approx \exists\beta.(\beta \to \alpha) \times \beta$$

A client that regards β as abstract succeeds:

$$\begin{array}{rl} & \langle\!\langle \mathit{unpack}_D\ M_1\ (\lambda(f,y).\ f\ y) : \tau \rangle\!\rangle \\ = & \exists\alpha.(\langle\!\langle M_1 : D\ \alpha \rangle\!\rangle \wedge \forall\beta.\langle\!\langle \lambda(f,y).\ f\ y : ((\beta \to \alpha) \times \beta) \to \tau \rangle\!\rangle) \\ \equiv & \exists\alpha.(\langle\!\langle M_1 : D\ \alpha \rangle\!\rangle \wedge \forall\beta.\ \mathit{def}\ f : \beta \to \alpha; y : \beta\ \mathit{in}\ \langle\!\langle f\ y : \tau \rangle\!\rangle) \\ \equiv & \exists\alpha.(\langle\!\langle M_1 : D\ \alpha \rangle\!\rangle \wedge \forall\beta.\tau = \alpha) \\ \equiv & \exists\alpha.(\langle\!\langle M_1 : D\ \alpha \rangle\!\rangle \wedge \tau = \alpha) \\ \equiv & \langle\!\langle M_1 : D\ \tau \rangle\!\rangle \end{array}$$

Existential types calls for universal types!

Exercise We reuse the type $D\ \alpha \approx \exists\beta.(\beta \to \alpha) \times \beta$ of frozen computations. Assume given a list l with elements of type $D\ \tau_1$.

Assume given a function g of type $\tau_1 \to \tau_2$. Transform the list into a new list l' of frozen computations of type $D\ \tau_2$ (without actually running any computation).

```
List.map (λ(z) let D(f, y) = z in D((λ(z) g (f z)), y))
```

Generalizing this example to a function that receives g and l and returns l' does not typecheck:

```
let lift g l =
  List.map (λ(z) let D(f, y) = z in D((λ(z) g (f z)), y))
```

In expression *let* α, x = *unpack* M_1 *in* M_2, occurrences of x in M_2 can only be passed to external functions (free variables) that are polymorphic so that x does not leak out of its context.

Uses of existential types

Mitchell and Plotkin [1988] note that existential types offer a means of explaining *abstract types.* For instance, the type:

$$\begin{aligned}\exists \mathit{stack}.\{&\mathit{empty} : \mathit{stack};\\ &\mathit{push} : \mathit{int} \times \mathit{stack} \to \mathit{stack};\\ &\mathit{pop} : \mathit{stack} \to \mathit{option}\ (\mathit{int} \times \mathit{stack})\}\end{aligned}$$

specifies an abstract implementation of integer stacks.

Unfortunately, it was soon noticed that the elimination rule is too awkward, and that existential types alone do not allow designing *module systems* [Harper and Pierce, 2005].

Montagu and Rémy [2009] make existential types *more flexible* in several important ways, and argue that they might explain modules after all.

Existential types in OCaml

Existential types are available indirectly in OCaml as a degenerate case of GADT and via abstract types and first-class modules.

Via GADT (iso-existential types)

```
type 'a d = D : ('b → 'a) * 'b → 'a d
let freeze f x = D (f, x)
let run (D (f, x)) = f x
```

Via first-class modules (abstract types)

```
module type D = sig type b type a val f : b → a val x : b end
let freeze (type u) (type v) f  x =
   (module struct type b =  u type a = v  let f = f let x = x end : D);;
let unfreeze (type u) (module M : D with type a = u) = M.f M.x
```

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Typed closure conversion

Everything is now set up to prove that, in System F with existential types:

$$\Gamma \vdash M : \tau \quad \text{implies} \quad [\![\Gamma]\!] \vdash [\![M]\!] : [\![\tau]\!]$$

Environment-passing closure conversion

Assume $\Gamma \vdash \lambda x.\, M : \tau_1 \to \tau_2$ and $\mathrm{dom}(\Gamma) = \{x_1, \ldots, x_n\} = \mathrm{fv}(\lambda x.\, M)$.

$$
\begin{array}{lll}
[\![\lambda x{:}\tau_1.\, M]\!] & = & \textit{let } code : \qquad\qquad\qquad\qquad = \\
 & & \quad \lambda(env : \quad , x : \quad). \\
 & & \quad\quad \textit{let } (x_1, \ldots, x_n : \quad) = env \textit{ in} \\
 & & \quad\quad [\![M]\!] \\
 & & \textit{in} \\
 & & \textit{pack} \quad , (code, (x_1, \ldots, x_n)) \\
 & & \textit{as}
\end{array}
$$

Environment-passing closure conversion

Assume $\Gamma \vdash \lambda x.\, M : \tau_1 \to \tau_2$ and $\mathrm{dom}(\Gamma) = \{x_1, \ldots, x_n\} = \mathrm{fv}(\lambda x.\, M)$.

$$
\begin{array}{lll}
[\![\lambda x{:}\tau_1.\, M]\!] & = & \mathit{let}\ \mathit{code} : \qquad\qquad\qquad\qquad = \\
 & & \quad \lambda(\mathit{env} : [\![\Gamma]\!], x : [\![\tau_1]\!]). \\
 & & \quad\quad \mathit{let}\ (x_1, \ldots, x_n : [\![\Gamma]\!]) = \mathit{env}\ \mathit{in} \\
 & & \quad\quad [\![M]\!] \\
 & & \mathit{in} \\
 & & \mathit{pack} \qquad , (\mathit{code}, (x_1, \ldots, x_n)) \\
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\begin{array}{lll}
[\![\lambda x{:}\tau_1.\, M]\!] & = & \mathit{let}\ \mathit{code} : ([\![\Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!] = \\
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& & \quad\quad [\![M]\!] \\
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& & \quad\quad \mathit{let}\ (x_1, \ldots, x_n : [\![\Gamma]\!]) = \mathit{env}\ \mathit{in} \\
& & \quad\quad [\![M]\!] \\
& & \mathit{in} \\
& & \mathit{pack}\ [\![\Gamma]\!], (\mathit{code}, (x_1, \ldots, x_n)) \\
& & \mathit{as}\ \exists\alpha.((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times \alpha
\end{array}
$$

Environment-passing closure conversion

Assume $\Gamma \vdash \lambda x.\, M : \tau_1 \to \tau_2$ and $\mathrm{dom}(\Gamma) = \{x_1, \ldots, x_n\} = \mathrm{fv}(\lambda x.\, M)$.

$$
\begin{array}{rcl}
[\![\lambda x{:}\tau_1.\, M]\!] & = & \mathit{let}\ \mathit{code} : ([\![\Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!] = \\
& & \quad \lambda(\mathit{env} : [\![\Gamma]\!], x : [\![\tau_1]\!]). \\
& & \quad\quad \mathit{let}\ (x_1, \ldots, x_n : [\![\Gamma]\!]) = \mathit{env}\ \mathit{in} \\
& & \quad\quad [\![M]\!] \\
& & \mathit{in} \\
& & \mathit{pack}\ [\![\Gamma]\!], (\mathit{code}, (x_1, \ldots, x_n)) \\
& & \mathit{as}\ \exists\alpha.((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times \alpha
\end{array}
$$

We find $[\![\Gamma]\!] \vdash [\![\lambda x{:}\tau_1.\, M]\!] : [\![\tau_1 \to \tau_2]\!]$, as desired.

Environment-passing closure conversion

Assume $\Gamma \vdash M : \tau_1 \to \tau_2$ and $\Gamma \vdash M_1 : \tau_1$.

$$\begin{aligned} [\![M\ M_1]\!] \quad = \quad & \mathit{let}\ \alpha, (\mathit{code} : (\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!], \mathit{env} : \alpha) = \\ & \quad \mathit{unpack}\ [\![M]\!]\ \mathit{in} \\ & \quad \mathit{code}\ (\mathit{env}, [\![M_1]\!]) \end{aligned}$$

We find $[\![\Gamma]\!] \vdash [\![M\ M_1]\!] : [\![\tau_2]\!]$, as desired.

Environment-passing closure conversion recursion

Recursive functions can be translated in this way, known as the "fix-code" variant [Morrisett and Harper, 1998] (leaving out type information):

$$
\begin{aligned}
[\![\mu f.\lambda x.M]\!] \quad = \quad & \textit{let rec } code\ (env, x) = \\
& \quad \textit{let } f = \textit{pack}\ (code, env)\ \textit{in} \\
& \quad \textit{let}\ (x_1, \ldots, x_n) = env\ \textit{in} \\
& \quad [\![M]\!]\ \textit{in} \\
& \textit{pack}\ (code, (x_1, \ldots, x_n))
\end{aligned}
$$

where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\mu f.\lambda x.M)$.

The translation of applications is unchanged: recursive and non-recursive functions have an identical calling convention.

What is the weak point of this variant?

Environment-passing closure conversion recursion

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where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\mu f.\lambda x.M)$.

The translation of applications is unchanged: recursive and non-recursive functions have an identical calling convention.

What is the weak point of this variant?

A new closure is allocated at every call.

Environment-passing closure conversion recursion

Instead, the "fix-pack" variant [Morrisett and Harper, 1998] uses an extra field in the environment to store a back pointer to the closure:

$$
\begin{array}{rcl}
[\![\mu f.\lambda x.M]\!] & = & \textit{let } code = \lambda(env, x). \\
& & \quad \textit{let } (f, x_1, \ldots, x_n) = env \textit{ in} \\
& & \quad [\![M]\!] \\
& & \textit{in} \\
& & \textit{let rec } clo = (code, (clo, x_1, \ldots, x_n)) \textit{ in} \\
& & clo
\end{array}
$$

where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\mu f.\lambda x.M)$.

This requires general, recursively-defined *values.* Closures are now *cyclic* data structures.

Environment-passing closure conversion — recursion

Here is how the "fix-pack" variant is type-checked. Assume $\Gamma \vdash \mu f.\lambda x.M : \tau_1 \to \tau_2$ and $\mathrm{dom}(\Gamma) = \{x_1, \ldots, x_n\} = \mathrm{fv}(\mu f.\lambda x.M)$.

$$
\begin{array}{l}
[\![\mu f \qquad .\lambda x.M]\!] = \\
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\quad\quad\quad [\![M]\!] \textit{ in} \\
\quad \textit{let rec } clo : \qquad = \\
\quad\quad \textit{pack} \qquad , (code, (clo, x_1, \ldots, x_n)) \\
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\end{array}
$$

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$$
\begin{array}{l}
[\![\mu f : \tau_1 \to \tau_2.\lambda x.M]\!] = \\
\quad \textit{let } code : \qquad\qquad = \\
\quad\quad \lambda(env : [\![f : \tau_1 \to \tau_2, \Gamma]\!], x : [\![\tau_1]\!]). \\
\quad\quad\quad \textit{let } (f, x_1, \dots, x_n) : [\![f : \tau_1 \to \tau_2, \Gamma]\!] = env \textit{ in} \\
\quad\quad\quad [\![M]\!] \textit{ in} \\
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\begin{array}{l}
[\![\mu f : \tau_1 \to \tau_2.\lambda x.M]\!] = \\
\quad \textit{let } code : ([\![f : \tau_1 \to \tau_2; \Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!] = \\
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Environment-passing closure conversion recursion

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\quad\quad \lambda(env : [\![f : \tau_1 \to \tau_2, \Gamma]\!], x : [\![\tau_1]\!]). \\
\quad\quad\quad \mathit{let}\ (f, x_1, \ldots, x_n) : [\![f : \tau_1 \to \tau_2, \Gamma]\!] = env\ \mathit{in} \\
\quad\quad\quad [\![M]\!]\ \mathit{in} \\
\quad \mathit{let\ rec}\ clo : [\![\tau_1 \to \tau_2]\!] = \\
\quad\quad \mathit{pack}\ [\![f : \tau_1 \to \tau_2, \Gamma]\!], (code, (clo, x_1, \ldots, x_n)) \\
\quad\quad\ \mathit{as}\ \exists\alpha((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times \alpha) \\
\quad \mathit{in}\ clo
\end{array}
$$

Environment-passing closure conversion recursion

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\quad\quad\quad \mathit{let}\ (f, x_1, \ldots, x_n) : [\![f : \tau_1 \to \tau_2, \Gamma]\!] = env\ \mathit{in} \\
\quad\quad\quad [\![M]\!]\ \mathit{in} \\
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\end{array}
$$

Problem?

Environment-passing closure conversion recursion

The recursive function may be polymorphic, but recursive calls are monomorphic...

We can generalize the encoding afterwards,

$$[\![\Lambda\vec{\beta}.\mu f : \tau_1 \to \tau_2.\lambda x.M]\!] = \Lambda\vec{\beta}.[\![\mu f : \tau_1 \to \tau_2.\lambda x.M]\!]$$

whenever the right-hand side is well-defined.

This allows the *indirect* compilation of polymorphic recursive functions as long as the recursion is monomorphic.

Fortunately, the encoding can be straightforwardly adapted to *directly* compile polymorphically recursive functions into polymorphic closure.

Environment-passing closure conversion recursion

$$
\begin{array}{l}
[\![\mu f : \forall\vec{\beta}.\,\tau_1 \to \tau_2.\,\lambda x.M]\!] = \\
\quad \mathit{let}\ code : \forall\vec{\beta}.\,([\![f : \forall\vec{\beta}.\,\tau_1 \to \tau_2; \Gamma]\!] \times [\![\tau_1]\!]) \to [\![\tau_2]\!] = \\
\quad\quad \lambda(env : [\![f : \forall\vec{\beta}.\,\tau_1 \to \tau_2, \Gamma]\!], x : [\![\tau_1]\!]). \\
\quad\quad\quad \mathit{let}\ (f, x_1, \ldots, x_n) : [\![f : \forall\vec{\beta}.\,\tau_1 \to \tau_2, \Gamma]\!] = env\ \mathit{in} \\
\quad\quad\quad [\![M]\!]\ \mathit{in} \\
\quad \mathit{let\ rec}\ clo : [\![\forall\vec{\beta}.\,\tau_1 \to \tau_2]\!] = \\
\quad\quad \Lambda\vec{\beta}.\mathit{pack}\ [\![f : \forall\vec{\beta}.\,\tau_1 \to \tau_2, \Gamma]\!], (code\ \vec{\beta}, (clo, x_1, \ldots, x_n)) \\
\quad\quad\quad \mathit{as}\ \exists\alpha((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]) \times \alpha) \\
\quad \mathit{in}\ clo
\end{array}
$$

The encoding is simple.

However, this requires the introduction of recursive non-functional values "let rec x = v". While this is a useful construct, it really alters the operational semantics and requires updating the type soundness proof.

- Introduction
- Towards typed closure conversion
- Existential types
  - Implicitly-type existential types passing
  - Iso-existential types
- Typed closure conversion
  - Environment passing
  - Closure passing

Closure-passing closure conversion

$$
\begin{aligned}
[\![\lambda x.\, M]\!] &= \mathit{let}\ code = \lambda(clo, x).\\
&\qquad \mathit{let}\ (\_, x_1, \ldots, x_n) = clo\ \mathit{in}\\
&\qquad [\![M]\!]\\
&\quad \mathit{in}\ (code, x_1, \ldots, x_n)\\
[\![M_1\ M_2]\!] &= \mathit{let}\ clo = [\![M_1]\!]\ \mathit{in}\\
&\quad \mathit{let}\ code = \mathit{proj}_0\ clo\ \mathit{in}\\
&\quad code\ (clo, [\![M_2]\!])
\end{aligned}
$$

where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\lambda x.\, M)$.

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&\qquad [\![M]\!]\\
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where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\lambda x.\, M)$.

How could we typecheck this? What are the difficulties?

Closure-passing closure conversion

$$
\begin{array}{rcl}
[\![\lambda x.\, M]\!] & = & \mathit{let}\ code = \lambda(clo, x).\\
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\end{array}
$$

There are two difficulties:

- a closure is a tuple, whose *first* field should be *exposed* (it is the code pointer), while the number and types of the remaining fields should be abstract;
- the first field of the closure contains a function that expects *the closure itself* as its first argument.

Closure-passing closure conversion

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What type-theoretic mechanisms could we use to describe this?

Closure-passing closure conversion

There are two difficulties:

- a closure is a tuple, whose *first* field should be *exposed* (it is the code pointer), while the number and types of the remaining fields should be abstract;
- the first field of the closure contains a function that expects *the closure itself* as its first argument.

What type-theoretic mechanisms could we use to describe this?

- existential quantification over the *tail* of a tuple (a.k.a. a *row*);
- *recursive types.*

Tuples, rows, row variables

The standard tuple types that we have used so far are:

$$\begin{array}{lcll} \tau & ::= & \ldots \mid \Pi\, R & \text{– types} \\ R & ::= & \epsilon \mid (\tau; R) & \text{– rows} \end{array}$$

The notation $(\tau_1 \times \ldots \times \tau_n)$ was sugar for $\Pi\,(\tau_1; \ldots; \tau_n; \epsilon)$.

Let us now introduce *row variables* and allow *quantification* over them:

$$\begin{array}{lcll} \tau & ::= & \ldots \mid \Pi\, R \mid \forall \rho.\, \tau \mid \exists \rho.\tau & \text{– types} \\ R & ::= & \rho \mid \epsilon \mid (\tau; R) & \text{– rows} \end{array}$$

This allows reasoning about the first few fields of a tuple whose length is not known.

Typing rules for tuples

The typing rules for tuple construction and deconstruction are:

$$\textsc{Tuple}\quad \frac{\forall i.\ \in [1,n] \quad \Gamma \vdash M_i : \tau_i}{\Gamma \vdash (M_1, \ldots, M_n) : \Pi\,(\tau_1; \ldots; \tau_n; \epsilon)} \qquad \textsc{Proj}\quad \frac{\Gamma \vdash M : \Pi\,(\tau_1; \ldots; \tau_i; R)}{\Gamma \vdash \mathit{proj}_i\ M : \tau_i}$$

These rules make sense with or without row variables

Projection does not care about the fields beyond i. Thanks to row variables, this can be expressed in terms of *parametric polymorphism:*

$$\mathit{proj}_i : \forall \alpha.\, {}_1 \ldots \alpha_i \rho.\, \Pi\,(\alpha_1; \ldots; \alpha_i; \rho) \to \alpha_i$$

About Rows

Rows were invented by Wand and improved by Rémy in order to ascribe precise types to operations on *records.*

The case of tuples, presented here, is simpler.

Rows are used to describe *objects* in Objective Caml [Rémy and Vouillon, 1998].

Rows are explained in depth by Pottier and Rémy [Pottier and Rémy, 2005].

Closure-passing closure conversion

Rows and recursive types allow to define the translation of types in the closure-passing variant:

$$
\begin{array}{lll}
 & [\![\tau_1 \to \tau_2]\!] & \\
= & \exists \rho. & \rho \text{ describes the environment} \\
 & \quad \mu\alpha. & \alpha \text{ is the concrete type of the closure} \\
 & \quad\quad \Pi\,(& \text{a tuple...} \\
 & \quad\quad\quad (\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]; & \text{...that begins with a code pointer...} \\
 & \quad\quad\quad \rho & \text{...and continues with the environment} \\
 & \quad\quad)
\end{array}
$$

See Morrisett and Harper's "fix-type" encoding [1998].

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Question: Why is it $\exists\rho.\ \mu\alpha.\ \tau$ and not $\mu\alpha.\ \exists\rho.\ \tau$

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Question: Why is it $\exists\rho.\ \mu\alpha.\ \tau$ and not $\mu\alpha.\ \exists\rho.\ \tau$

The type of the environment is fixed once for all and does not change at each recursive call.

Closure-passing closure conversion

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Question: Notice that ρ appears only once. Any comments?

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Usually, an existential type variable appears both at positive and negative occurrences.

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\end{array}
$$

See Morrisett and Harper's "fix-type" encoding [1998].

Question: Notice that ρ appears only once. Any comments?

Usually, an existential type variable appears both at positive and negative occurrences. Here, the variable appear only at a negative occurrence, but in a recursive part of the type that can be unfolded.

Closure-passing closure conversion

Let $Clo(R)$ abbreviate $\mu\alpha.\Pi\,((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]; R)$.

Let $UClo(R)$ abbreviate its unfolded version,
$\Pi\,((Clo(R) \times [\![\tau_1]\!]) \to [\![\tau_2]\!]; R)$.

We have $[\![\tau_1 \to \tau_2]\!] = \exists\rho.Clo(\rho)$.

$$
\begin{array}{lcl}
[\![\lambda x{:}\quad .M]\!] & = & \textit{let } code : \qquad\qquad\qquad\qquad = \\
 & & \quad \lambda(clo : \qquad\quad , x : \qquad). \\
 & & \quad\quad \textit{let } (\_, x_1, \ldots, x_n) : \qquad\quad = \textit{unfold } clo \textit{ in} \\
 & & \quad\quad [\![M]\!] \textit{ in} \\
 & & \textit{pack} \qquad , (\textit{fold } (code, x_1, \ldots, x_n)) \\
 & & \textit{as} \\
 & & \\
[\![M_1\ M_2]\!] & = & \textit{let } \rho, clo = \textit{unpack } [\![M_1]\!] \textit{ in} \\
 & & \textit{let } code : \qquad\qquad\qquad = \\
 & & \quad \textit{proj}_0\ (\textit{unfold } clo) \textit{ in} \\
 & & code\ (clo, [\![M_2]\!])
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$$

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$$
\begin{array}{rcl}
[\![\lambda x{:}[\![\tau_1]\!].\, M]\!] & = & \mathit{let}\ code : (Clo([\![\Gamma]\!]) \times [\![\tau_1]\!]) \to [\![\tau_2]\!] = \\
& & \quad \lambda(clo : Clo([\![\Gamma]\!], x : [\![\tau_1]\!]). \\
& & \quad\quad \mathit{let}\ (\_, x_1, \ldots, x_n) : UClo[\![\Gamma]\!] = \mathit{unfold}\ clo\ \mathit{in} \\
& & \quad\quad [\![M]\!]\ \mathit{in} \\
& & \mathit{pack}\ [\![\Gamma]\!], (\mathit{fold}\ (code, x_1, \ldots, x_n)) \\
& & \mathit{as}\ \exists\rho.\, Clo(\rho)
\end{array}
$$

$$
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[\![M_1\ M_2]\!] & = & \mathit{let}\ \rho, clo = \mathit{unpack}\ [\![M_1]\!]\ \mathit{in} \\
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& & \quad \mathit{proj}_0\ (\mathit{unfold}\ clo)\ \mathit{in} \\
& & code\ (clo, [\![M_2]\!])
\end{array}
$$

Closure-passing closure conversion recursive functions

In the closure-passing variant, recursive functions can be translated as:

$$
\begin{aligned}
[\![\mu f.\lambda x.M]\!] \quad = \quad & \mathit{let}\ code = \lambda(clo, x).\\
& \quad \mathit{let}\ f = clo\ \mathit{in}\\
& \quad \mathit{let}\ (\_, x_1, \ldots, x_n) = clo\ \mathit{in}\\
& \quad [\![M]\!]\\
& \mathit{in}\ (code, x_1, \ldots, x_n)
\end{aligned}
$$

where $\{x_1, \ldots, x_n\} = \mathrm{fv}(\mu f.\lambda x.M)$.

No extra field or extra work is required to store or construct a representation of the free variable f: the closure itself plays this role.

However, this untyped code can only be typechecked when recursion is monomorphic.

Exercise:

Check well-typedness with monomorphic recursion.

Closure-passing closure conversion recursive functions

The problem to adapt this encoding to polymorphic recursion is that recursive occurrences of f are rebuilt from the current invocation of the closure, *i.e.* is monomorphic since the closure is invoked after type specialization.

By contrast, in the environment passing encoding, the environment contained a polymorphic binding for the recursive calls that was filled with the closure before its invokation, *i.e.* with a polymorphic type.

Fortunately, we may slightly change the encoding, using a recursive closure as in the type-passing version, to allow typechecking in System F.

Closure-passing closure conversion recursive functions

Let τ be $\forall\vec{\alpha}.\,\tau_1 \to \tau_2$ and Γ_f be $f : \tau, \Gamma$ where $\vec{\beta} \# \Gamma$

$$
\begin{aligned}
[\![\mu f{:}\tau.\,\lambda x.M]\!] = \mathit{let}\ code = \\
&\Lambda\vec{\beta}.\lambda(clo : Clo[\![\Gamma_f]\!], x : [\![\tau_1]\!]). \\
&\quad \mathit{let}\ (\_code, f, x_1, \ldots, x_n) : \forall\vec{\beta}.\,UClo([\![\Gamma_f]\!]) = \\
&\qquad \mathit{unfold}\ clo\ \mathit{in} \\
&\quad [\![M]\!]\ \mathit{in} \\
&\mathit{let\ rec}\ clo : \forall\vec{\beta}.\,\exists\rho.\,Clo(\rho) = \Lambda\vec{\beta}. \\
&\quad \mathit{pack}\ [\![\Gamma]\!], (\mathit{fold}\ (code\ \vec{\beta}, clo, x_1, \ldots, x_n))\ \mathit{as}\ \exists\rho.\,Clo(\rho) \\
&\mathit{in}\ clo
\end{aligned}
$$

Remind that $Clo(R)$ abbreviates $\mu\alpha.\Pi\,((\alpha \times [\![\tau_1]\!]) \to [\![\tau_2]\!]; R)$. Hence, $\vec{\beta}$ are free variables of $Clo(R)$.

Here, a polymorphic recursive function is *directly* compiled into a polymorphic recursive closure. Notice that the type of closures is unchanged, so the encoding of applications is also unchanged.

Mutually recursive functions — Environment passing

Can we compile mutually recursive functions?

$$M \triangleq \mu(f_1, f_2).(\lambda x_1.\, M_1, \lambda x_2.\, M_2)$$

Environment passing:

$$[\![M]\!] \quad =$$

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Environment passing:

$$\begin{array}{lll}
[\![M]\!] & = & \textit{let } code_i = \lambda(env, x). \\
& & \quad \textit{let } (f_1, f_2, x_1, \ldots, x_n) = env \textit{ in} \\
& & \quad [\![M_i]\!] \\
& & \textit{in} \\
& & \textit{let rec } clo_1 = (code_1, (clo_1, clo_2, x_1, \ldots, x_n)) \\
& & \quad \textit{and } clo_2 = (code_2, (clo_1, clo_2, x_1, \ldots, x_n)) \textit{ in} \\
& & clo_1, clo_2
\end{array}$$

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Comments?

Mutually recursive functions — Environment passing

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Environment passing:

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& & \quad \textit{let } (f_1, f_2, x_1, \ldots, x_n) = env \textit{ in} \\
& & \quad [\![M_i]\!] \\
& & \textit{in} \\
& & \textit{let rec } env = (clo_1, clo_2, x_1, \ldots, x_n) \\
& & \quad \textit{and } clo_1 = (code_1, env) \\
& & \quad \textit{and } clo_2 = (code_2, env) \textit{ in} \\
& & clo_1, clo_2
\end{array}$$

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$$\begin{array}{l}
\textit{let } code_i = \lambda(clo, x). \\
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\textit{in} \\
\textit{let rec } clo_1 = (code_1, clo_1, clo_2, x_1, \ldots, x_n) \\
\quad \textit{and } clo_2 = (code_2, clo_1, clo_2, x_1, \ldots, x_n) \\
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\quad \textit{and } clo_2 = (code_2, clo_1, clo_2, x_1, \ldots, x_n) \\
\textit{in } clo_1, clo_2
\end{array}$$

Question: Can we share the closures c_1 and c_2 in case n is large?

Mutually recursive functions — Closure passing

Can we compile mutually recursive functions?

$$M \triangleq \mu(f_1, f_2).(\lambda x_1.\, M_1, \lambda x_2.\, M_2)$$

Closure passing:

let $code_1 = \lambda(clo, x).$
  let $(\_code_1, \_code_2, f_1, f_2, x_1, \ldots, x_n) = clo$ *in* $[\![M_1]\!]$ *in*
let $code_2 = \lambda(clo, x).$
  let $(\_code_2, f_1, f_2, x_1, \ldots, x_n) = clo$ *in* $[\![M_2]\!]$ *in*
let rec $clo_1 = (code_1, code_2, clo_1, clo_2, x_1, \ldots, x_n)$ *and* $clo_2 = c_1.tail$
in clo_1, clo_2

- $clo_1.tail$ returns a pointer to the tail $(code_2, clo_1, clo_2, x_1, \ldots, x_n)$ of clo_1 without allocating a new tuple.
- This is only possible with some support from the GC (and extra-complexity and runtime cost for GC)

Optimizing representations

Can closure passing and environment passing be mixed?

Optimizing representations

Can closure passing and environment passing be mixed?

No because the calling-convention (*i.e.*, the encoding of application) must be uniform.

However, their is some flexibility in the representation of the closure. For instance, the following change is completely local:

$$
\begin{array}{rcl}
[\![\lambda x.\, M]\!] & = & \mathit{let}\ code = \lambda(clo, x).\\
& & \quad \mathit{let}\ (\_, \quad x_1, \ldots, x_n \quad) = clo\ \mathit{in}\ [\![M]\!]\ \mathit{in}\\
& & \quad (code, \quad x_1, \ldots, x_n \quad)\\
[\![M_1\ M_2]\!] & = & \mathit{let}\ clo = [\![M_1]\!]\ \mathit{in}\\
& & \mathit{let}\ code = \mathit{proj}_0\ clo\ \mathit{in}\\
& & code\ (clo, [\![M_2]\!])
\end{array}
$$

Applications? When many definitions share the same closure, the closure (or part of it) may be shared.

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& & (code,\ (\ x_1, \ldots, x_n\ )\ )\\
[\![M_1\ M_2]\!] & = & \mathit{let}\ clo = [\![M_1]\!]\ \mathit{in}\\
& & \mathit{let}\ code = \mathit{proj}_0\ clo\ \mathit{in}\\
& & code\ (clo, [\![M_2]\!])
\end{array}
$$

Applications? When many definitions share the same closure, the closure (or part of it) may be shared.

Encoding of objects

The closure-passing representation of mutually recursive functions is similar to the representations of objects in the object-as-record-of-functions paradigm:

A class definition is an object generator:

$$
\begin{array}{l}
class\ c\ (x_1, \ldots x_q)\{ \\
\quad \textit{meth}\ \ m_1 = M_1 \\
\quad \ldots \\
\quad \textit{meth}\ \ m_p = M_p \\
\}
\end{array}
$$

Given arguments for parameter $x_1, \ldots x_1$, it will build recursive methods $m_1, \ldots m_n$.

Encoding of objects

A class can be compiled into an object closure:

$$
\begin{array}{l}
\mathit{let}\ m = \\
\qquad \mathit{let}\ m_1 = \lambda(m, x_1, \ldots, x_q).\, M_1\ \mathit{in} \\
\qquad \ldots \\
\qquad \mathit{let}\ m_p = \lambda(m, x_1, \ldots, x_q).\, M_p\ \mathit{in} \\
\qquad \{m_1, \ldots, m_p\}\ \ \mathit{in} \\
\lambda x_1 \ldots x_q.\, (m, x_1, \ldots x_q)
\end{array}
$$

Each m_i is bound to the code for the corresponding method.
The code of all methods are combined into a record of methods, which is shared between all objects of the same class.

Calling method m_i of an object p is

$$(\mathit{proj}_0\ p).m_i\ p$$

How can we type the encoding?

Typed encoding of objects

Let τ_i be the type of M_i, and row R describe the types of $(x_1, \ldots x_q)$.

Let $Clo(R)$ be $\mu\alpha.\Pi(\{(m_i : \alpha \to \tau_i)^{i\in 1..n}\}; R)$ and $UClo(R)$ its unfolding.

Fields R are hidden in an existential type $\exists\rho.\, \mu\alpha.\Pi(\{(m_i : \alpha \to \tau_i)^{i\in I}\}; \rho)$:

$$
\begin{array}{l}
\mathit{let}\ m = \{ \\
\quad m_1 = \lambda(m, x_1, \ldots x_q : UClo(R)).\, [\![M_1]\!] \\
\quad \ldots \\
\quad m_p = \lambda(m, x_1, \ldots x_q : UClo(R)).\, [\![M_p]\!] \\
\}\ \mathit{in} \\
\lambda x_1. \ldots \lambda x_q.\, \mathit{pack}\ R, \mathit{fold}\ (m, x_1, \ldots x_q)\ \mathit{as}\ \exists\rho.\, (M, \rho)
\end{array}
$$

Calling a method of an object p of type M is

$$p\#m_i \triangleq \mathit{let}\ \rho, z = \mathit{unpack}\ p\ \mathit{in}\ (\mathit{proj}_0\ \mathit{unfold}\ z).m_i\ z$$

An object has a recursive type but it is *not* a recursive value.

Typed encoding of objects

Typed encoding of objects were first studied in the 90's to understand what objects really are in a type setting.

These encodings are in fact type-preserving compilation of (primitive) objects.

There are several variations on these encodings. See [Bruce et al., 1999] for a comparison.

See [Rémy, 1994] for an encoding of objects in (a small extension of) ML with iso-existentials and universals.

See [Abadi and Cardelli, 1996, 1995] for more details on primitive objects.

Moral of the story

Type-preserving compilation is rather *fun.* (Yes, really!)

It forces compiler writers to make the structure of the compiled program *fully explicit,* in type-theoretic terms.

In practice, building explicit type derivations, ensuring that they remain small and can be efficiently typechecked, can be a lot of work.

Optimizations

Because we have focused on type preservation, we have studied only naïve closure conversion algorithms.

More ambitious versions of closure conversion require program analysis: see, for instance, Steckler and Wand [1997]. These versions *can* be made type-preserving.

Other challenges

Defunctionalization, an alternative to closure conversion, offers an interesting challenge, with a simple solution [Pottier and Gauthier, 2006].

Designing an efficient, type-preserving compiler for an *object-oriented language* is quite challenging. See, for instance, Chen and Tarditi [2005].

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(Most titles have a clickable mark "▷" that links to online versions.)

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