1 Introduction

2 Splitting unpack

3 Splitting pack

4 Reduction

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Abstract

We present a variant of the explicitly-typed second-order polymorphic \( \lambda \)-calculus with primitive open existential types, i.e. a collection of more atomic constructs for introduction and elimination of existential types. We equip the language with a call-by-value small-step reduction semantics that enjoys the subject reduction property.

We claim that open existential types model abstract types and type generativity in a modular way. Our proposal can be understood as a logically-motivated variant of Dreyer’s RTG where type generativity is no more seen as a side effect.
Open Existential types for Module systems
A Logical Account of Type Generativity

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Based on joint work with

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Motivations

Modular programming is the key to writing good, maintainable software. Will be even more important tomorrow than today.

However, despite 20 years of intensive research on module systems: There is a big gap between:

- The intuitive simplicity of the underlying concepts, and
- The actual complexity of existing solutions.

Our goals

- Explain or reduce this gap.
- Design a core calculus for the surface language of a language with:
  - first-class modules
  - that is conceptually economical, e.g. avoids duplication of concepts.
What is needed for module systems?

Already in the core-calculus

- Structures are records
- Functors are functions
- Signatures are types

Crucial (and deep) features for expressiveness

- Type abstraction (may already be in the core language)
- Type generativity (the master-key to modules)

Important (but not so deep) features for conciseness

- Sharing a posteriori (diamond import problem)
- Flexible naming policy
Type generativity

The problem

- A defines t abstractly
- B and C uses A
- Can D assume that B and C have compatible views of t?
- Can also two copies/views of A be made incompatible? —this is type generativity.

Keep track of identifies of abstract types in a way or another
Previous approaches

Existential types: model type abstraction but lack modular structure.

Path-based systems.
- An old idea (Dave MacQueen, Modules for Standard ML, 1984)
- Today, still at the basis of all module systems.

General idea
- Cannot refer to how types have been defined, since they have been forgotten.
- Instead refer to where they have been defined.
- An abstract type is referred to as a projection path from a value variable.
Problem with path-based systems

General problem

- Types depend on values (at least syntactically)
- Although paths only use a small fragment of dependent types, a much larger fragment is needed to preserve stability under term substitution.

Dependent types

- An overkill technology.
- They do not carry good intuitions about modules (in our opinion).
- Too complicated to be exposed to the programmer, hence they defined a core calculus in which existing languages are elaborated.

Elaboration semantics

- Elaboration is a compilation process, may be of arbitrary complexity.
- The user cannot perform it mentally.
- Looses the connection with logic: no small-step reduction semantics.
Dreyer’s RTG: a solution without dependent types!

**Motivations**
- Designed and used as an *internal language*
- for a language with *recursive* and *mixin* modules.

**Underlying ideas**
- Sees type generativity as a *static* side effect.
- Use of linear types to keep track of such side effects.

**Achievements**
- Interesting set of primitives
- which can be used to model recursive and mixin modules.
- Type generativity can be explained without dependent types.
Problem with $\text{RTG}$

**Based on and carrying wrong intuitions**

- **Type generativity is a side effect** (claimed very strongly)
- Their semantics enforces and relies on a strictly deterministic evaluation order.

**Ad hoc meta-theory**

- Typechecking in $\text{RTG}$ uses an abstract machine that performs side effects into a global store.
- Their dynamic semantics is store based, including the modelling of generativity.

**Consequences**

- Unintuitive semantics: programmers can’t run the machine mentally.
- Any connection with logic is lost.
- Cannot be exposed to users, *i.e.* used as an external language.
F.zip: a variant of RTG without the drawbacks

Standard static and dynamic semantics

- Typing rules are compositional and have a logical flavor.
- Small-step reduction semantics
- The two are related by subject reduction and progress lemmas.
- No use of recursive types is needed to model type generativity (but they could be useful with recursive or mixin modules)

Curry-Howard isomorphism (for a subset of F.zip)

- Formulae are the same as in System-F with existential types.
- The same formulae are provable.
- There are more proofs—which can be assembled more modularly.
- Reduction is proof normalization, indeed.
Modules can be explained as a combination of

- open abstract types, to model type generativity
- *Shape bounded quantification* to recover conciseness

*(complementary, not described here)*
Reminder: pack and unpack

**Pack**

\[ \Gamma \vdash M : \tau'[\alpha \leftarrow \tau] \]

\[ \Gamma \vdash \text{pack } \langle \tau, M \rangle \text{ as } \exists \alpha. \tau' : \exists \alpha. \tau' \]

**Unpack**

\[ \Gamma \vdash M : \exists \alpha. \tau \quad \Gamma, \alpha, x : \tau \vdash M' : \tau' \quad \alpha \notin \text{ftv}(\tau') \]

\[ \Gamma \vdash \text{unpack } M \text{ as } \alpha, x \text{ in } M' : \tau' \]
Splitting unpack

unpack \( M \) as \( \alpha, x \) in \( M' \)

\[ \nu \alpha. \quad \text{let } x = \quad \text{open } \langle \alpha \rangle M \quad \text{in } M' \]

- Limits the scope of \( \alpha \)
- Binds \( M \) to \( x \) in \( M' \)
- Uses \( \alpha \) for the abstract type of \( M \)
\[ \nu \alpha. \quad \text{let } x = \quad \text{open } \langle \alpha \rangle M \quad \text{in } M' \]
Splitting unpack

\[ \nu \alpha. \quad \text{let } x = D \left\{ \begin{array}{l} \text{open } \langle \alpha \rangle M \\ \end{array} \right\} \text{ in } M' \]

\( M \) need not be at toplevel.
$\nu \alpha. \ {C \Bigg \{ \begin{array}{c}
\text{let } x = \text{open } \langle \alpha \rangle \ M \\
\text{in } M' \end{array} \Bigg \}}$

$\alpha$ need not be hidden immediately.
Splitting unpack

\[ C \left\{ \begin{array}{l}
\text{let } x = \text{open } \langle \alpha \rangle M \text{ in } M'
\end{array} \right\} \]

\( \alpha \) need not be hidden at all in program \textit{components}
Typechecking

Must forbid incorrect programs such as

\[
\nu \alpha. \quad \text{let } x = \text{open } \langle \alpha \rangle \ M_1 \text{ in } \text{open } \langle \alpha \rangle \ M_2 \text{ X ok } M_1[\alpha] \ x
\]
Typechecking
Typechecking

\[
\begin{align*}
\text{Nu} \\
\Gamma, \exists \alpha \vdash M : \tau & \quad \alpha \not\in \text{ftv}(\tau) \\
\hline
\Gamma \vdash \nu \alpha. M : \tau
\end{align*}
\]
Typechecking

\[ \text{OPEN} \]

\[ \Gamma \vdash M : \exists \alpha. \tau \]

\[ \Gamma, \exists \alpha \vdash \text{open} \langle \alpha \rangle M : \tau \]

\[ \text{NU} \]

\[ \Gamma, \exists \alpha \vdash M : \tau \quad \alpha \notin \text{ftv}(\tau) \]

\[ \Gamma \vdash \nu \alpha. M : \tau \]
Typechecking

\[ \text{Open} \]
\[ \Gamma \vdash M : \exists \alpha. \tau \]
\[ \Gamma, \exists \alpha \vdash \text{open}\langle \alpha \rangle M : \tau \]

\[ \text{Let} \]
\[ \Gamma_1 \vdash M_1 : \tau_1 \quad \Gamma_2, x : \tau_1 \vdash M_2 : \tau_2 \]
\[ \Gamma_1 \upharpoonright \Gamma_2 \vdash \text{let } x = M_1 \text{ in } M_2 : \tau_2 \]

\[ \nu \text{U} \]
\[ \Gamma, \exists \alpha \vdash M : \tau \quad \alpha \notin \text{ftv}(\tau) \]
\[ \Gamma \vdash \nu \alpha. M : \tau \]
Typechecking

\[ \text{OPEN} \]

\[ \Gamma \vdash M : \exists \alpha. \tau \]

\[ \Gamma, \exists \alpha \vdash \text{open} \langle \alpha \rangle M : \tau \]

\[ \Gamma, \forall \alpha \vdash M'[\alpha] : \tau' \]

\[ \text{LET} \]

\[ \Gamma_1 \vdash M_1 : \tau_1 \quad \Gamma_2, x : \tau_1 \vdash M_2 : \tau_2 \]

\[ \Gamma_1 \upharpoonright \Gamma_2 \vdash \text{let } x = M_1 \text{ in } M_2 : \tau_2 \]

\[ \text{NU} \]

\[ \Gamma, \exists \alpha \vdash M : \tau \quad \alpha \notin \text{ftv}(\tau) \]

\[ \Gamma \vdash \nu \alpha. M : \tau \]
Zipping of two type environments ensures that every existential type appears in at most one of the environments.

\[
\forall \alpha \sqsubseteq \forall \alpha = \forall \alpha \\
\forall \alpha \sqsubseteq \exists \alpha = \exists \alpha \\
\exists \alpha \sqsubseteq \forall \alpha = \exists \alpha
\]

\[
x : \tau \sqsubseteq x : \tau = x : \tau
\]

\[
\emptyset \sqsubseteq \emptyset = \emptyset
\]

\[
(\Gamma_1, b_1) \sqsubseteq (\Gamma_2, b_2) = (\Gamma_1 \sqsubseteq \Gamma_2), (b_1 \sqsubseteq b_2)
\]

\[
b ::= x : \tau \mid \forall \alpha \mid \exists \alpha\]
Splitting pack

pack \langle \tau, M \rangle \text{ as } \exists \alpha. \tau'

makes \alpha abstract with witness \tau

converts the type of \( M \) using the equation(s)
Splitting pack

pack $\langle \tau, M \rangle$ as $\exists \alpha. \tau'$

\[ \triangleq \]

$\exists \beta. \Sigma \langle \beta \rangle (\alpha = \tau) (M : \tau')$

closes the abstract type $\beta$

defines the open abstract type $\beta$

with internal name $\alpha$ and witness $\tau$

converts the type of $M$
Splitting pack

$$\text{pack } \langle \tau, M \rangle \text{ as } \exists \alpha. \tau'$$

$$\triangleq$$

$$\exists \beta. \ C \left\{ \Sigma \langle \beta \rangle (\alpha = \tau) \ D\{ (M : \tau') \} \right\}$$
Splitting pack

\[ \text{pack } \langle \tau, M \rangle \text{ as } \exists \alpha. \tau' \]

\[ \triangleq \]

\[ \sum \langle \beta \rangle (\alpha = \tau) \quad D\{ (M : \tau') \} \]

A module with an open abstract type \( \beta \).
Splitting pack

\[
C \left\{ \sum \langle \beta \rangle (\alpha = \tau) \right. \\
D \left\{ (M : \tau') \right. \\
A \text{ sub-module with an open abstract type } \beta.
\]
Typechecking

\[ \exists \beta. M : \forall \beta. \tau \]

\[ \Gamma, \exists \beta \vdash M : \tau \]

\[ \Gamma \vdash \exists \beta. M : \exists \beta. \tau \]
Typechecking

\[
\exists \beta \vdash M : \tau \\
\exists \beta. M : \exists \beta. \tau \\
\Gamma, \exists \beta \vdash \text{open } \langle \beta \rangle M : \tau
\]
Typechecking

Sigma

\[ \Gamma, \forall \beta, \Gamma', \forall (\alpha = \tau) \vdash M : \tau' \]

\[ \Gamma, \exists \beta, \Gamma' \vdash \Sigma \langle \beta \rangle (\alpha = \tau) M : \tau'[\alpha \leftarrow \beta] \]

Exists

\[ \Gamma, \exists \beta \vdash M : \tau \]

\[ \Gamma \vdash \exists \beta. M : \exists \beta. \tau \]

Open

\[ \Gamma \vdash M : \exists \beta. \tau \]

\[ \Gamma, \exists \beta \vdash \text{open} \langle \beta \rangle M : \tau \]
Typechecking

\[
\text{COERCION}
\]
\[
\Gamma \vdash M : \tau' \quad \Gamma \vdash \tau' \equiv \tau
\]
\[
\Gamma \vdash (M : \tau) : \tau
\]

\[
\text{SIGMA}
\]
\[
\Gamma, \forall \beta, \Gamma', \forall (\alpha = \tau) \vdash M : \tau'
\]
\[
\Gamma, \exists \beta, \Gamma' \vdash \Sigma\langle \beta\rangle (\alpha = \tau) M : \tau'[\alpha \leftarrow \beta]
\]

\[
\text{EXISTS}
\]
\[
\Gamma, \exists \beta \vdash M : \tau
\]
\[
\Gamma \vdash \exists \beta. M : \exists \beta. \tau
\]
Summary

Types are unchanged

\[ \tau ::= \alpha \mid \tau \to \tau \mid \forall \alpha. \tau \mid \exists \alpha. \tau \]

Expressions are

\[ M ::= \ldots \mid \exists \alpha. M \mid \sum \langle \beta \rangle (\alpha = \tau) M \mid (M : \tau) \mid \nu \alpha. M \mid \text{open} \langle \alpha \rangle M \]
Examples

In ML:

\[
\text{module } X = \text{struct} \begin{cases}
\text{type } t = \text{int} \\
\text{val } z = 0 \\
\text{val } s = \lambda (x : \text{int}) x + 1
\end{cases} : \text{sig} \begin{cases}
\text{type } t \\
\text{val } z : t \\
\text{val } s : t \to t
\end{cases}
\]

In Fzip:

\[
\Sigma \langle \beta \rangle (\alpha = \text{int}) \begin{cases}
\{
\text{z = 0 ;}
\text{s = } \lambda (x : \text{int}) x + 1
\} : \{
\text{z : } \alpha ;
\text{s : } \alpha \to \alpha
\}
\end{cases}
\]
Examples

In ML:

\[
\text{module } X = \text{struct}
\begin{align*}
\text{type } t &= \text{int} \\
\text{val } z &= 0 \\
\text{val } s &= \lambda(x : \text{int})x + 1
\end{align*}
\text{: sig}
\begin{align*}
\text{type } t \\
\text{val } z : t \\
\text{val } s : t \to t
\end{align*}
\]

In Fzip:

\[
\text{let } x = \exists(\alpha = \text{int}) \left( \begin{cases} 
z &= 0 \\
\lambda(x : \text{int})x + 1
\end{cases} \right) : \begin{cases} 
z : \alpha \\
\lambda : \alpha \to \alpha
\end{cases} \text{ in } \text{open } \langle \beta \rangle x
\]

Examples

In ML:

Making generative views of $x$

In Fzip:

let $x = \exists (\alpha = \text{int}) \left( \begin{cases} z = 0 ; \\ s = \lambda(x : \text{int})x + 1 \end{cases} \right) : \left( \begin{cases} z : \alpha ; \\ s : \alpha \rightarrow \alpha \end{cases} \right)$ in

let $x_1 = \text{open} \langle \beta_1 \rangle x$ in
let $x_2 = \text{open} \langle \beta_2 \rangle x$ in
\ldots
Examples

Functors

- functions must be pure (i.e. not create open abstract types)
- thus, body of functors are \textit{closed} abstract types
- that are opened after each application of the functor.

Example

\begin{verbatim}
let MakeSet = \\
  \Lambda \alpha. \lambda (cmp : \alpha \to \alpha \to bool) \exists (\beta = set(\alpha)) (\ldots : set(\beta)) in \\
let s_1 = open \langle \beta_1 \rangle MakeSet [int] (<) in \\
let s_2 = open \langle \beta_2 \rangle MakeSet [\beta_1] (s_1.cmp) in \\
\ldots
\end{verbatim}
Reduction

Problem (well-known)
- Expressions that create open abstract types can’t be substituted.
- This would duplicate—hence break—the use of linear resources.
- The reduct would thus be ill-typed.

Solution (new)
- Extrude $\Sigma$’s whenever needed (when reduction would blocked).
- This safely enlarges the scope of identities,
- moving the $\Sigma$’s outside of redexes, and
- Allowing further reduction to proceed.
Reduction

Example

\[
\text{let } x = \sum \langle \beta \rangle (\alpha = \text{int}) (1 : \alpha) \text{ in } \{ \ell_1 = x ; \ell_2 = (\lambda (y : \beta)y) x \}
\]

\[
\downarrow
\]

\[
\sum \langle \beta \rangle (\alpha = \text{int}) \text{ let } x = (1 : \alpha) \text{ in } \{ \ell_1 = x ; \ell_2 = (\lambda (y : \beta)y) x \}
\]

\[
\downarrow
\]

\[
\sum \langle \beta \rangle (\alpha = \text{int}) \{ \ell_1 = (1 : \alpha) ; \ell_2 = (\lambda (y : \beta)y) (1 : \alpha) \}
\]

\[
\downarrow
\]

\[
\sum \langle \beta \rangle (\alpha = \text{int}) \{ \ell_1 = (1 : \alpha) ; \ell_2 = (1 : \alpha) \}
\]
Reduction

- Results are non erroneous expressions that cannot be reduced.
- Some results cannot be duplicated and are not values.
- Values are results that can be duplicated.

Definition

Values

\[ \begin{align*}
  v &::= u \mid (u : \tau) \\
  u &::= x \mid \lambda(x : \tau)M \mid \Lambda\alpha. M \mid \exists\beta. \Sigma\langle\beta\rangle(\alpha = \tau)v
\end{align*} \]

Results

\[ \begin{align*}
  w &::= v \mid \Sigma\langle\beta\rangle(\alpha = \tau)w
\end{align*} \]

Note

- Abstractions \(\lambda's\) and \(\Lambda's\) are always values because they are pure, i.e. typechecked in \(\Gamma\) without \(\exists\alpha's\).
- Otherwise, unpure abstractions should be treated linearly.
Reduction

Call-by-value small-step reduction semantics

Elimination rules: $\beta$-reduction rules plus,

- $\text{open} \langle \beta \rangle \exists \alpha. M \leadsto M[\alpha \leftarrow \beta]$
- $\nu \beta. \Sigma \langle \beta \rangle (\alpha = \tau) w \leadsto w[\beta \leftarrow \alpha][\alpha \leftarrow \tau]$

+ Extrusion rule applies for all extrusion contexts $E$ (definition omitted)

+ Propagation of coercions (uninteresting reduction rules)
Theorem (Subject reduction)

If $\Gamma \vdash M : \tau$ and $M \leadsto M'$, then $\Gamma \vdash M' : \tau$.

Theorem (Progress)

If $\Gamma \vdash M : \tau$ and $\Gamma$ does not contain value variable bindings, then either $M$ is a result, or it is reducible.
The appearance of recursive types

Internal recursion, through openings:

\[
\text{let } x = \exists (\alpha = \beta \to \beta) \, M \text{ in open } \langle \beta \rangle \, x
\]

reduces to:

\[
\text{open } \langle \beta \rangle \, \exists (\alpha = \beta \to \beta) \, M
\]

which leads to the recursive equation \( \beta = \beta \to \beta \).

External recursion, through open witness definitions:

\[
\{ \ell_1 = \Sigma \langle \beta_1 \rangle (\alpha_1 = \beta_2 \to \beta_2) \, M_1 ; \\
\ell_2 = \Sigma \langle \beta_2 \rangle (\alpha_2 = \beta_1 \to \beta_1) \, M_2 \}
\]

already contains the recursive equations \( \beta_1 = \beta_2 \to \beta_2 \) and \( \beta_2 = \beta_1 \to \beta_1 \)

Cannot occur in System F.
The appearance of recursive types

Origin of the problem

\[ \Sigma \]

\[ \Gamma, \forall \beta, \Gamma', \forall (\alpha = \tau) \vdash M : \tau' \]

\[ \Gamma, \exists \beta, \Gamma' \vdash \Sigma \langle \beta \rangle (\alpha = \tau) M : \tau'[\alpha \leftarrow \beta] \]

\( \beta \) may appear in \( \tau \) which is later meant to be equated with \( \beta \).

Solutions

1. Remove \( \forall \beta \) from the premise:
   - requires that \( \Gamma' \) does not depend on \( \beta \) either.
   - too strong:
     - at least requires some special case for let-bindings.
     - some useful cases would still be eliminated.

2. Keep a more precise track of dependencies.
Tracking dependencies

**Traditional view**

- $\Gamma$ is a mapping together with a total ordering on its domain.

**Generalization**

- Organize the context as a strict partial order.
Relation to System F (with pack and unpack)

There is a subset $F^{\forall^-}$ with more restrictive dependencies

- System F is a subset of $F^{\forall^-}$
- There is a translation of pure expressions of $F^{\forall^-}$ to System F that
  - preserves the semantics, abstraction, and typings.
  - preserves $\beta$-reduction steps, but increases $let$-reduction steps.

Reading through the Curry-Howard isomorphism for $F^{\forall^-}$

- The formulae are the same as in System F.
- The provable formulae are the same as in System F.
- They are more proofs in $F^{\forall^-}$, which can be assembled in mode modular ways.
Conclusions

Type generativity can be explained by open existential types
- Standard small step reduction semantics.
  Scope extrusion is a good, fine grain explanation of type abstraction
- Linearity provides a good explanation of type generativity.
- Close connection to logic with new ways of assembling proofs.

Modelling of double-vision is already in $F^\forall$ (omitted)

Extension to recursive values and types (with no expected difficulties)

Shapes bounded polymorphism and projections (complementary)

Good basis for a core calculus for a rich surface language with
- first-class, recursive and mixin modules and no redundancies.
Appendix

7 Dependencies

8 Double vision

9 Related works
Tracking dependencies

Traditional view

- $\Gamma$ is a mapping together with a total ordering on its domain.

Generalization

- Organize the context as a strict partial order.
- $\Gamma$ is a pair $(\mathcal{E}, \prec)$ where $\mathcal{E}$ is a set of bindings ordered by $\prec$.
- We write $\Gamma, (b \prec D), \Gamma'$ when $\text{dom } \Gamma \not\prec b$ and $b \not\prec \text{dom } \Gamma'$ and $D$ is the set $b$ depends on.

Zipping of contexts is redefined

- $(\mathcal{E}_1, \prec_1) \sqcup (\mathcal{E}_2, \prec_2) = ((\mathcal{E}_1 \sqcup \mathcal{E}_2), (\prec_1 \cup \prec_2)^+)$
- $\mathcal{E}_1 \sqcup \mathcal{E}_2 = \{b_1 \sqcup b_2 \mid b_1 \in \mathcal{E}_1, b_2 \in \mathcal{E}_2, \text{dom } b_1 = \text{dom } b_2\}$
  $\cup \{\exists \beta \mid \beta \in \text{dom } \mathcal{E}_1 \Delta \text{dom } \mathcal{E}_2\}$
  (weakening to remove unnecessary dependencies)
Tracking dependencies

**Sigma**

\[
\mathcal{D}' \setminus (\{\beta\} \cup \text{dom} \Gamma') \subseteq \mathcal{D}
\]

\[
\Gamma, (\forall \beta \in \mathcal{D}), \Gamma', (\forall (\alpha = \tau') \in \mathcal{D}') \vdash M : \tau
\]

\[
\Gamma, (\exists \beta \in \mathcal{D}), \Gamma' \vdash \Sigma \langle \beta \rangle (\alpha = \tau') M : \tau[\alpha \leftarrow \beta]
\]

In particular,

- Free variables of the witness type \(\tau'\) are in \(\mathcal{D}'\) (by well-formedness).
- Those that are in \(\text{dom} \Gamma\) are not in \(\text{dom} \Gamma'\) and thus must be in \(\mathcal{D}\).
Tracking dependencies

\[ \Sigma \]

\[ D' \setminus (\{ \beta \} \cup \text{dom} \Gamma') \subseteq D \]

\[ \Gamma', (\forall \beta \not\in D), (\forall (\alpha = \tau') \not\in D') \vdash M : \tau \]

\[ \Gamma, (\exists \beta \not\in D), \Gamma' \vdash \Sigma \langle \beta \rangle (\alpha = \tau') M : \tau[\alpha \leftarrow \beta] \]

Prevents typechecking:

\[ \{ \ell_1 = \Sigma \langle \beta_1 \rangle (\alpha_1 = \beta_2 \rightarrow \beta_2) M_1 ; \quad \text{implies} \quad \beta_1 \prec \beta_2 \}
\]

\[ \ell_2 = \Sigma \langle \beta_2 \rangle (\alpha_2 = \beta_1 \rightarrow \beta_1) M_2 \} \quad \text{implies} \quad \beta_2 \prec \beta_1 \]

But allows typechecking:

\[ \{ \ell_1 = \Sigma \langle \beta_1 \rangle (\alpha_1 = \text{int}) M_1 ; \]

\[ \ell_2 = \Sigma \langle \beta_2 \rangle (\alpha_2 = \beta_1 \rightarrow \beta_1) M_2 \} \]
Tracking dependencies

\[
\text{OPEN}
\]

\[
\Gamma \vdash M : \exists \beta. \tau \quad \mathcal{D} = \text{dom} \Gamma
\]

\[
\Gamma, (\exists \beta \in \mathcal{D}) \vdash \text{open} \langle \beta \rangle M : \tau
\]

\[
\text{LET}
\]

\[
\{ \alpha \mid (\exists \alpha) \in \Gamma_2 \text{ and } (\forall \alpha) \in \Gamma_1 \} \subseteq \mathcal{D}
\]

\[
\Gamma_1 \vdash M_1 : \tau_1 \quad \Gamma_2, (x : \tau_1 \in \mathcal{D}) \vdash M_2 : \tau_2
\]

\[
\Gamma_1 \not\subseteq \Gamma_2 \vdash \text{let } x = M_1 \text{ in } M_2 : \tau_2
\]

**Open:** \( \alpha \) depends on all that precedes him, since the witness is unknown.

**Let:** \( x \) depends on all abstract types that are used in \( M_2 \) and could be seen in \( M_1 \).
Tracking dependencies

**Open**

\[
\frac{\Gamma \vdash M : \exists \beta. \tau}{\Gamma, (\exists \beta \in \mathcal{D}) \vdash \text{open} \langle \beta \rangle M : \tau}
\]

\[
\mathcal{D} = \text{dom } \Gamma
\]

**Let**

\[
\{ \alpha \mid (\exists \alpha) \in \Gamma_2 \text{ and } (\forall \alpha) \in \Gamma_1 \} \subseteq \mathcal{D}
\]

\[
\frac{\Gamma_1 \vdash M_1 : \tau_1 \quad \Gamma_2, (x : \tau_1 \in \mathcal{D}) \vdash M_2 : \tau_2}{\Gamma_1 \notackcross \Gamma_2 \vdash \text{let } x = M_1 \text{ in } M_2 : \tau_2}
\]

Prevents typechecking:

\[
\text{let } x = \exists (\alpha = \beta \rightarrow \beta) M \text{ in open } \langle \beta \rangle x
\]

imply \( x \prec \beta \), since \( \beta \in \text{dom } \Gamma_2 \)

imply \( \beta \prec x \)
Double vision

This example is rejected

\[ \text{let } f = \lambda(x : \beta)x \text{ in } \Sigma \langle \beta \rangle (\alpha = \text{int}) f (1 : \alpha) \]

*We do not know that the external type \( \beta \) in the type of \( f \) is equal to the internal view \( \alpha \) also equal to int.*

Keep this information in the context

\[
\Sigma \quad \Gamma, \forall \alpha, \Gamma', \forall (\alpha \triangleleft \beta = \tau') \vdash M : \tau
\]

\[
\Gamma, \exists \beta, \Gamma' \vdash \Sigma \langle \beta \rangle (\alpha = \tau') M : \tau[\alpha \leftarrow \beta]
\]

and use it whenever needed

\[
\Sigma \quad \Gamma \vdash M : \tau' \quad \Gamma \vdash \tau \triangleleft \tau'
\]

\[
\Gamma \vdash M : \tau
\]
Comparisson with Derek’s RTG

The primitives are similar, with small differences

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<tr>
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<th>RTG</th>
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<td>new $\alpha$ in $M$</td>
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<tr>
<td>$\Sigma \langle \alpha \rangle \ (\alpha = \tau) \ M$</td>
<td>set $\alpha := \tau$ in $M$</td>
</tr>
<tr>
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<td>$\Lambda \alpha \uparrow K. \lambda (\ : () \ 1 \ M$</td>
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- We evaluate under existentials while RTG does not.
- RTG uses $F^\omega$ while we restrict to System F.
- RTG allows recursive values and types, while we do not.
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- We evaluate under existentials while RTG does not.
- RTG uses $\text{F}^\omega$ while we restrict to System F.
- RTG allows recursive values and types, while we do not.

Shared ideas with RTG

- Use of linear types
  (only in typing contexts in Fzip, exposed in RTG.)
- Similar decomposition of constructs
  (by design in Fzip, observed a posteriori in RTG.)
Comparissson with Derek’s RTG

The primitives are similar, with small differences

Shared ideas with RTG

The “inside” differs significantly

- Typechecking in RTG uses an abstract machine that performs side effects into a global store.
- Unintuitive for programmers (who can’t run the machine mentally).
- Looses the connection with logic.
- Does not isolate type abstraction from the use of recursive types.

The motivations and uses also differs

- Designed and used as an internal language (opposite to our goals)
- Used to model recursive and mixin modules (complementary)
Other related works

Rossberg (2003)
Introduces $\lambda_N$, a version of System-F to define abstract types, that can automatically be extruded to allow sharper type analysis.

- Many similarities in spirit with our $\Sigma$ binder.
- But the motivations and technical details are quite different. In particular, parametricity is purposely violated in $\lambda_N$.

Russo (2003)
- He first explained that paths are meaningless for module types.
- He interpretes modules and signatures into semantic objets within $F^\omega$.
- However
  - his existential types are implicitly opened.
  - no dynamic semantics for objets.