A Work-Efficient Algorithm for Parallel Unordered Depth-First Search

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High-performance graph traversal

- In a *graph traversal*, computation proceeds from one vertex to the next through the edges in the graph.
- Improved performance for graph traversal means improved performance for many other algorithms.
- The main challenge is coping with irregularity in graphs.
- In this work, we present a new algorithm
  - to perform fast traversal over large, in-memory directed graphs
  - using a (single, dedicated) multicore system
  - achieving:
    - analytical bounds showing work-efficiency and high-parallelism, and
    - an implementation that outperforms state-of-the-art codes (almost always)

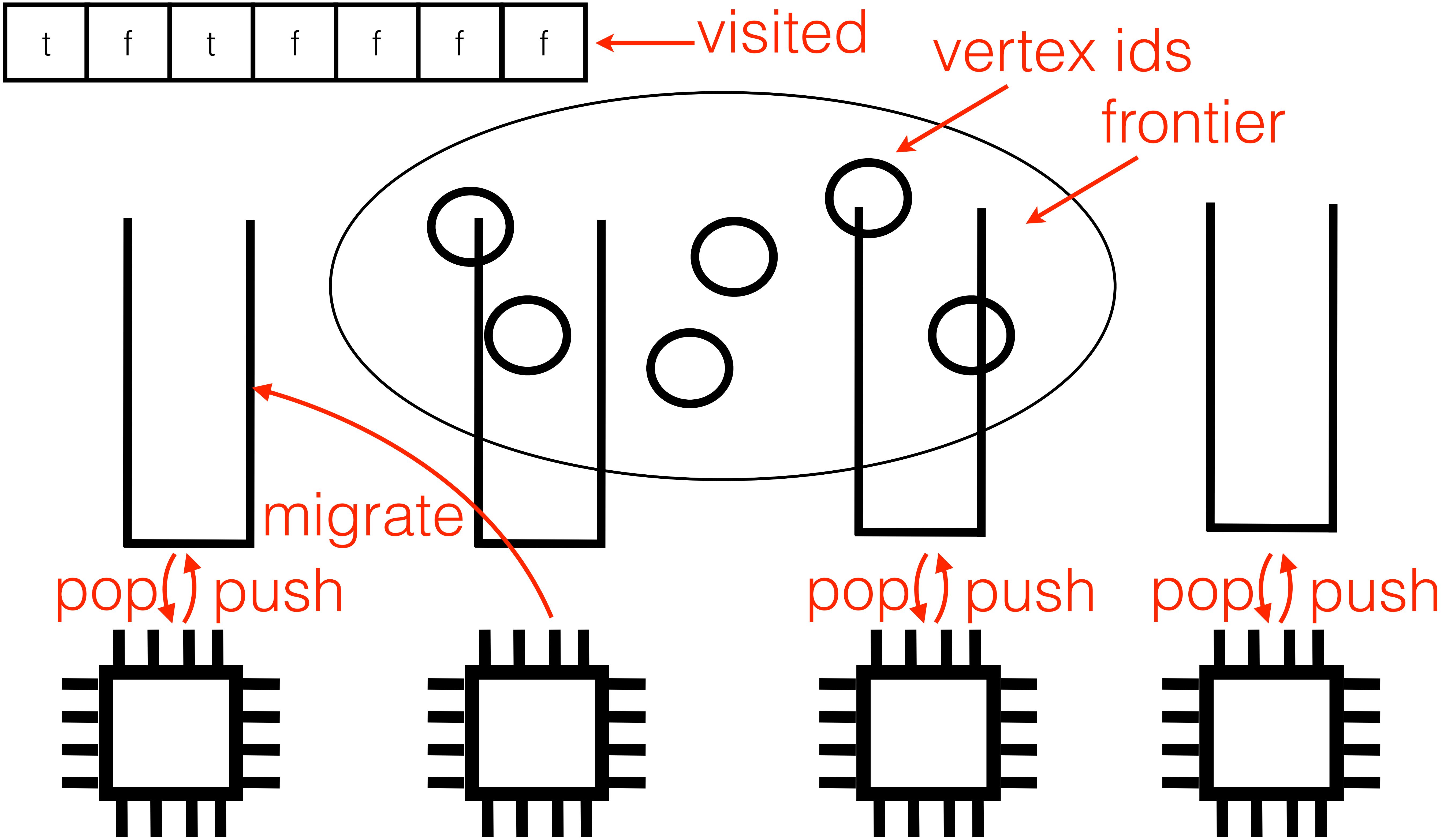
Motivation

- Most of the recent attention in the research literature on graph traversal is paid to parallel BFS.
- Why parallel BFS but not parallel DFS?
  - Parallel DFS with strict ordering is known to be P-complete (i.e., hard to parallelize).
- However, loosely ordered, parallel DFS:
  - relaxes the strict DFS ordering slightly
  - achieves a high degree of parallelism
  - has many applications, e.g.,
    - reachability analysis & graph search
    - parallel garbage collection (Jones et al 2011), etc…
    - KLA graph-processing framework (Harshvardhan et al 2014)
- When feasible, Pseudo DFS is preferred because it is usually faster than the alternatives.

Pseudo DFS (PDFS)

- Input:
  - directed graph and ID of source vertex
- Output:
  - the set of vertices connected by a path to the source vertex

PDFS



t	f	t	f	f	f	f

visited

vertex ids

frontier

migrate

pop push

pop push

pop push

PDFS vs. PBFS

Synchronization

- PDFS is *asynchronous*:
  - Each core traverses independently from its frontier.
- PBFS is *level synchronous*:
  - Cores traverse the graph level by level, in lock step, synchronizing between every two levels.

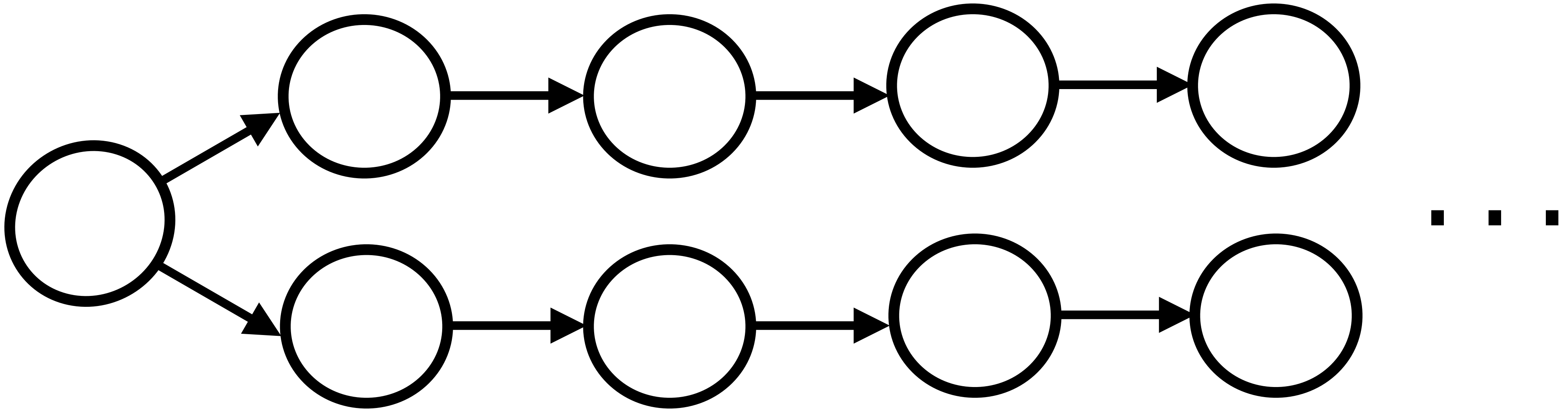
Data locality

- DFS is preferred in parallel GC.
  - e.g., mark sweep
- Why?
  - DFS visits heap objects in the order in which objects were allocated.

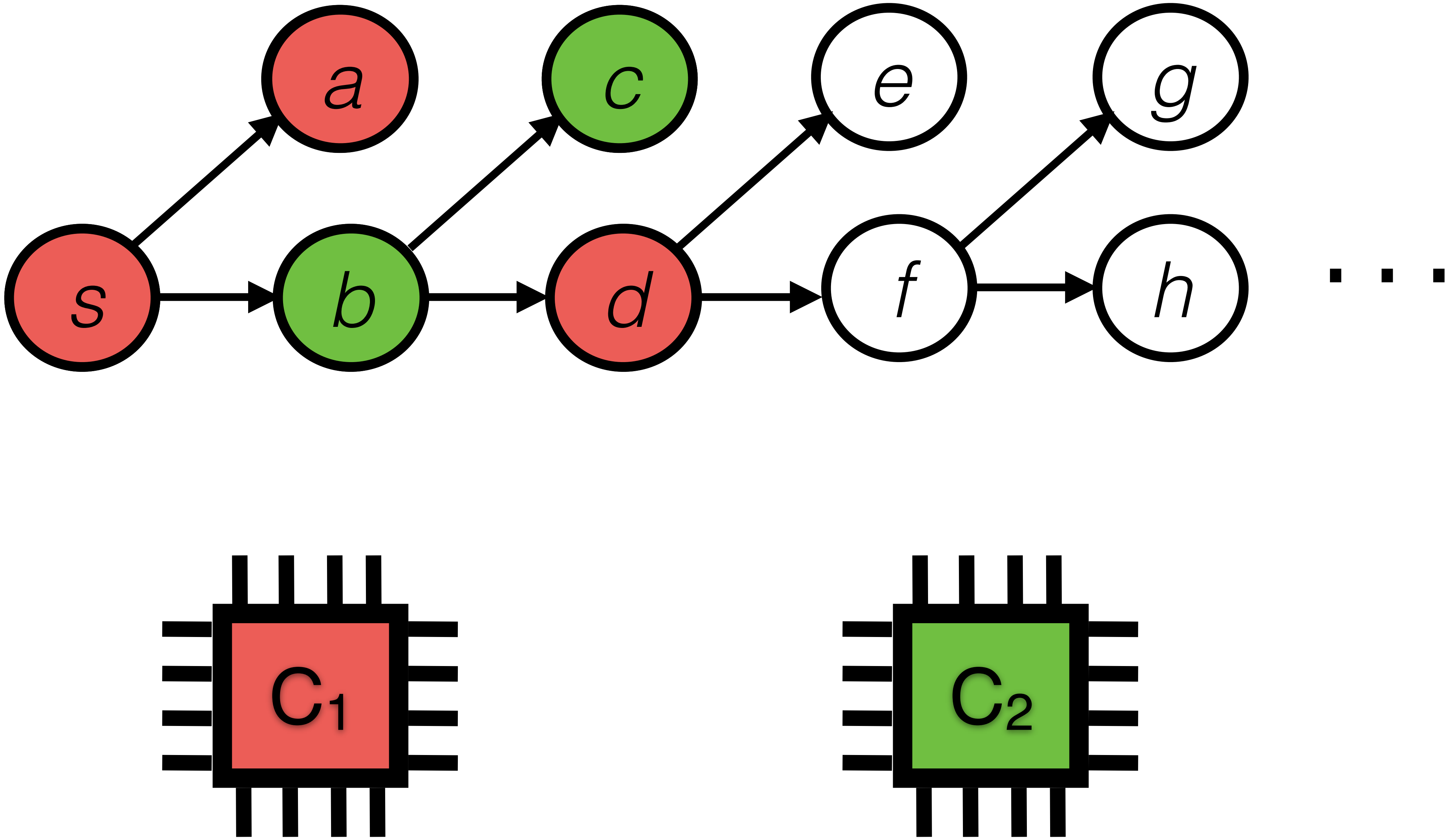
The granularity-control challenge

- The key tradeoff is between:
  - the cost to pay for migrating some chunk of work, and
  - the benefit of parallelizing the migrated work
- Migrate too often, it’s too slow; too infrequently, it’s too slow.
- Granularity control is a particular challenge for PDFS because, when you migrate a piece of frontier, you have little information about how much work you’re giving away.

Example in favor of aggressively sharing work



Example against sharing work



Granularity control by batching vertices

- A *batch* is a small, fixed-capacity buffer that stores part of the frontier.
- In batching, each work-stealing queue stores pointers to batches of vertices.
- Idea: use batches to amortize the cost of migrating work.
- Previous state of the art for PDFS:
    - Batching PDFS (Cong et al 2008)
    - Parallel mark-sweep GC (Endo 1997 and Seibert 2010)
- No batching PDFS so far guarantees against worst-case behavior.

Our work

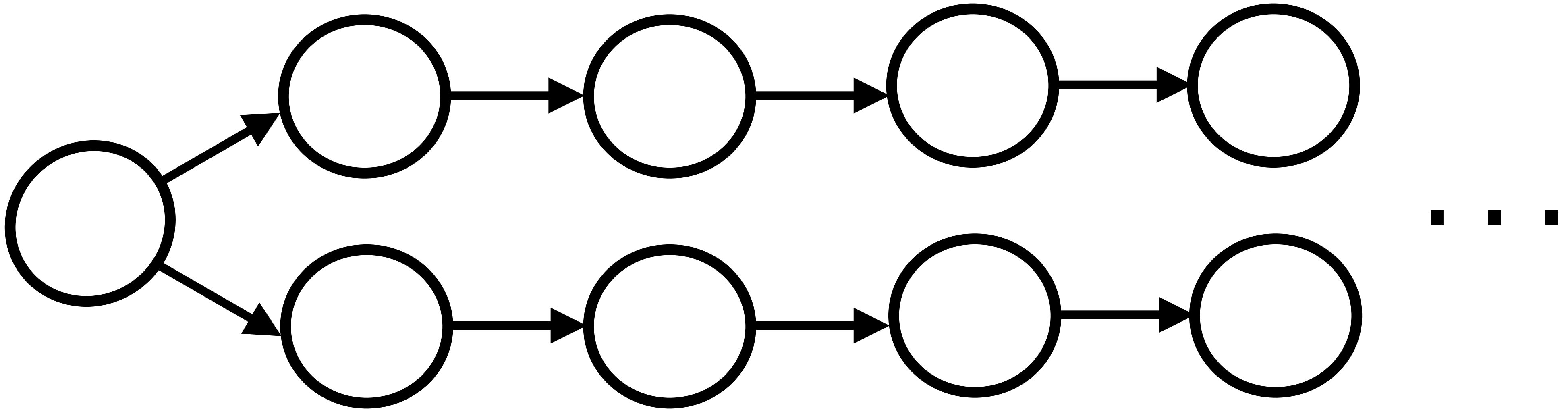
Central question:
Can we bring to PDFS the analytical and empirical rigor that has been applied to PBFS, but keep the benefits of a DFS-like traversal?

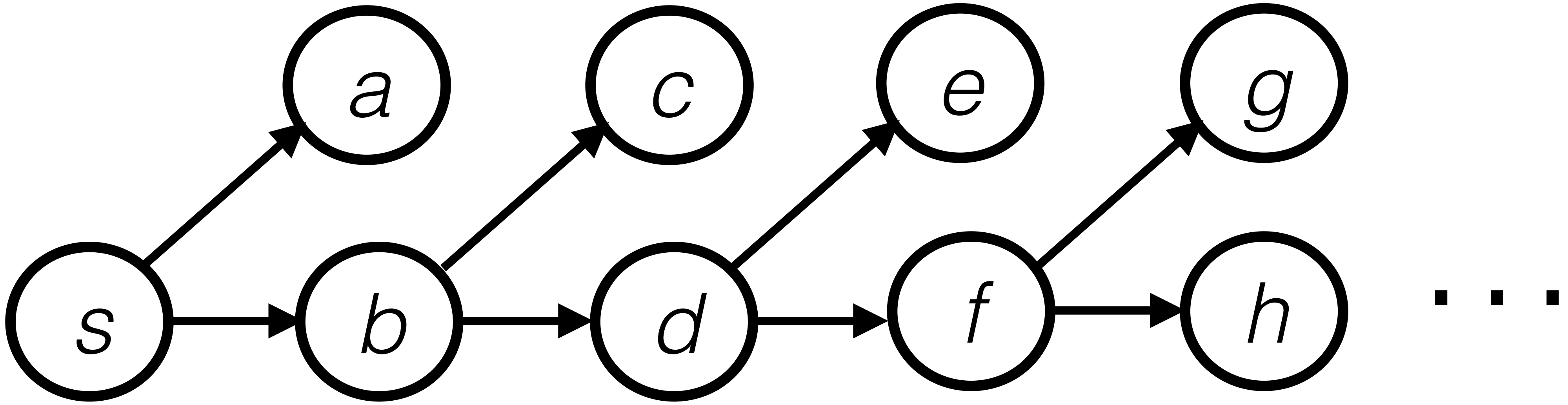
- We present a new PDFS algorithm.
- In a realistic cost model:
  - We show that our PDFS is *work efficient*:
    - Running time on a single core is the same as that of serial DFS, up to constant factors.
  - We show that our PDFS is highly parallel.
- In experiments on a machine with 40 cores, we show the following.
  - Our PDFS outperforms alternative algorithms across many of a varied set of input graphs.
  - Our PDFS can exploit data locality like sequential DFS.

Our solution to granularity control

- Migration of work is realized by message passing.
  - Each core regularly polls the status of a cell (in RAM).
  - When core C_1 requests work from C_2, C_1 writes its ID into the cell owned by C_2.
  - Each core owns a private frontier.
- Our granularity control technique: when receiving a query, a core shares its frontier only if one of the following two conditions is met:
  - The frontier is larger than some fixed constant, K.
  - The core has treated at least K edges already
- The setting for K can be picked once based (solely) on the characteristics of the machine.

Why is our granularity-control technique effective?





Our PDFS algorithm

Tuning parameters:

- *K*: positive integer controlling the eagerness of work sharing
- *D*: positive integer controlling the frequency of polling

Each core does:

- if my frontier is empty
  - repeatedly query random cores until finding work
- else
  - handle an incoming request for work
  - process up to *D* edges:
    - for each edge ending at vertex *v*
      - if this core wins the race to claim *v*, push outgoing neighbors of *v* into the frontier
      - remove *v* from the frontier

To handle a work request, a core does:

- if frontier contains at least *K* edges or has at least two edges and has treated at least *K* edges since previously sending work:
  - transfer half of the local frontier to the frontier of the hungry core
- notify the hungry core

Analytical bounds

Theorem 1
The number of migrations is $3m/K$.

Shows that each work migration is amortized over at least $K/3$ edges.

Theorem 2
The total amount of work performed is linear in the size of the input graph.

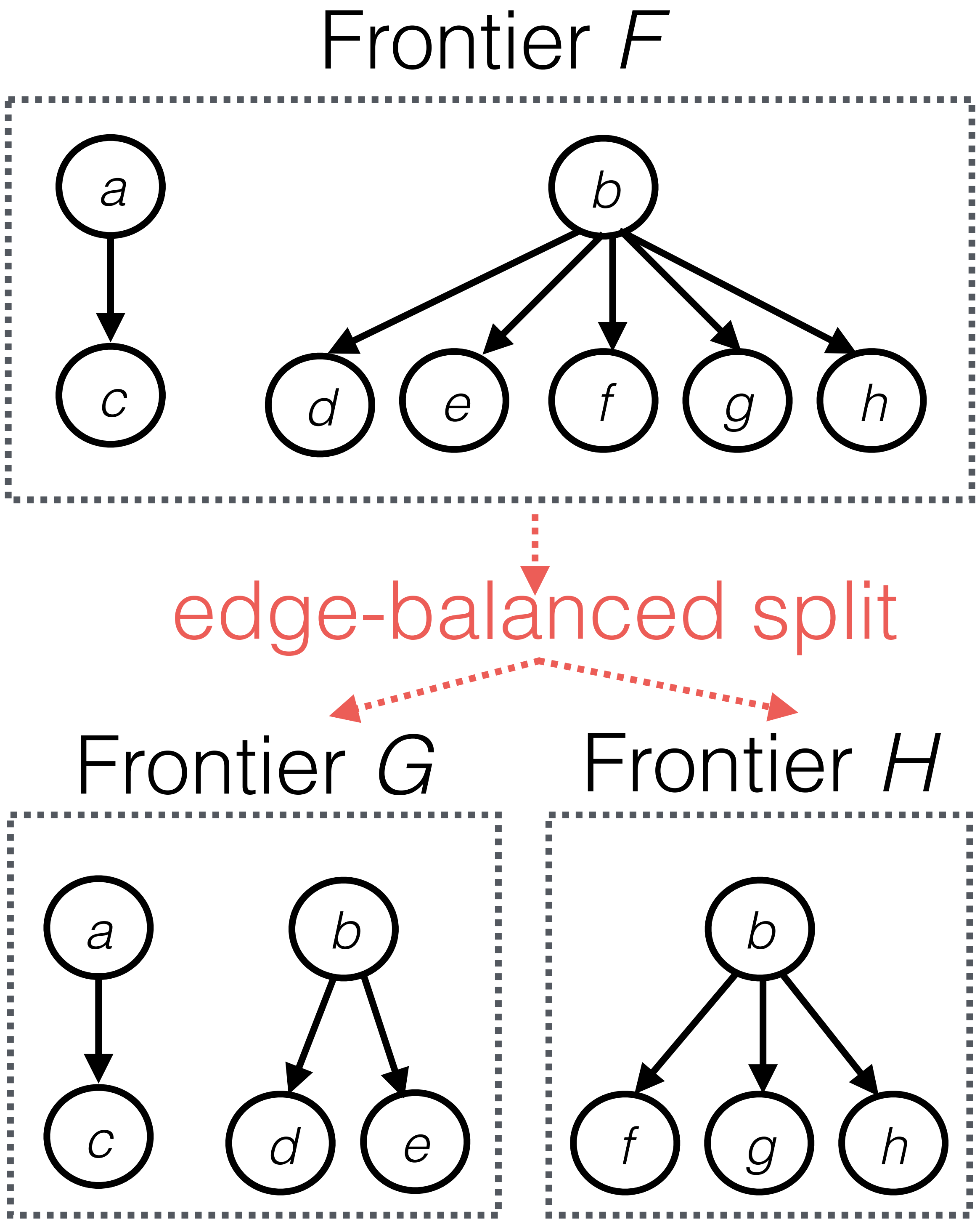
Shows that all polling and communication costs are well amortized.

Theorem 3
Each work query is matched by a response in $O(D + \log n)$ time.

Shows that the algorithm can achieve almost every opportunity for parallelism.

Our frontier data structure

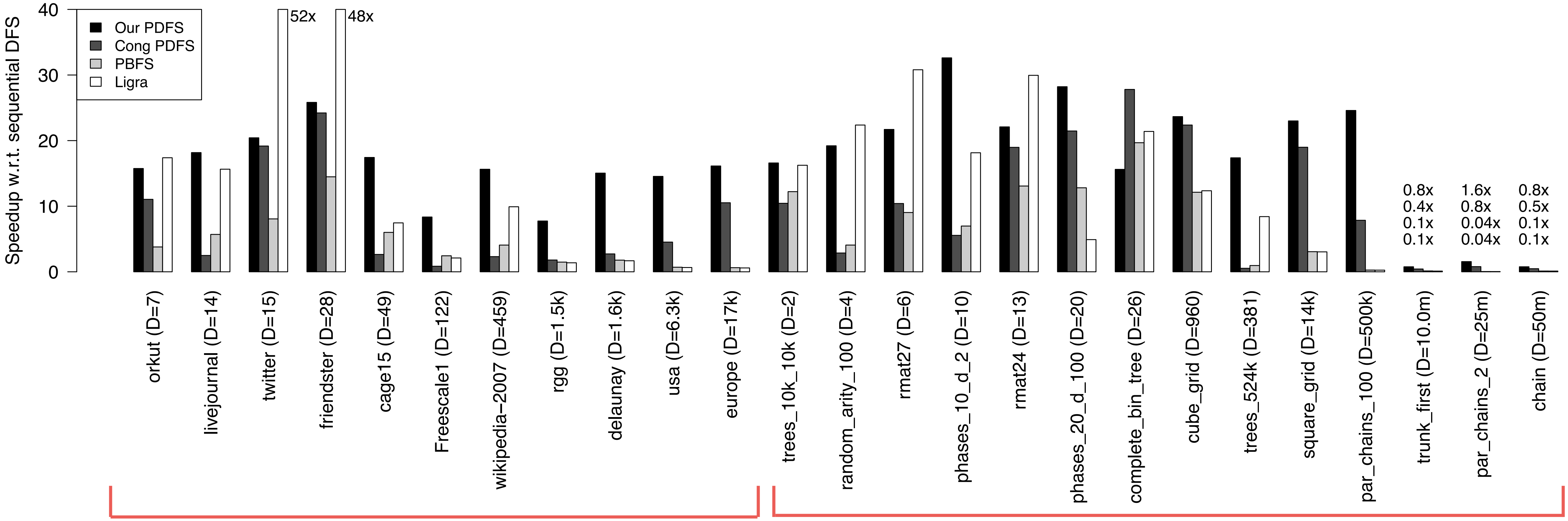
- It is based on our previous work on a chunked-tree data structure.
- It's a sequence data structure storing weighted items.
- It can
  - push/pop in constant time
  - split in half according to the weights of the items in logarithmic time.
- In the PDFS frontier, a weight represents the outdegree of a vertex.
- It enables:
  - rapidly migrating large chunks of frontier on the fly
  - efficiently parallelizing high-outdegree vertices



Experimental results

higher = better

- 40 Xeon cores @ 2.4Ghz
- 1 TB RAM



real world

synthetic

diameter D

diameter D

Related work

- PDFS
  - Batching PDFS (Cong et al 2008)
  - Parallel mark-sweep GC (Endo 1997 and Seibert 2010)
- PBFS
  - Work-efficient Parallel BFS (Leiserson & Schardl 2010)
  - Direction-optimizing BFS (Beamer et al 2012)
  - Ligra (Shun & Blelloch 2013)
- Hybrid PDFS/PBFS
  - KLA graph-processing framework (Harshvardhan et al 2014)

Summary

- We presented a new PDFS algorithm.
- Our results lift PDFS to a level of rigor similar to that of work-efficient PBFS.
- In our paper:
  - We show that PDFS exploits data locality as effectively as serial DFS.
- Our results show that PDFS performs well both in theory and practice.
- The results suggest that our PDFS may be useful as a component of other algorithms and graph-processing systems.