Fork-join model and work stealing

CRSPP reading group

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MPI-SWS

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Outline

- Background: design and analysis of parallel algorithms
- Scheduling parallel algorithms on multiprocessor machines
- Scheduling by work stealing
- Implementation of work stealing

Implicit parallelism

Divide and conquer style is typical for parallel algorithms.

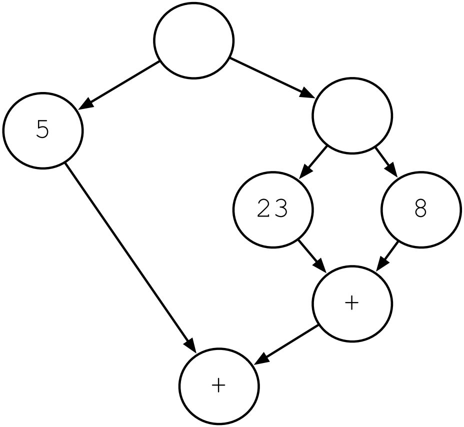
```
type tree =
  | Leaf of int
  | Node of tree * tree

let rec sum t =
  match t with
  | Leaf n -> n
  | Node (t1,t2) ->
    (* allow recursive calls to go in parallel *)
    let (n1, n2) = (| sum t1, sum t2 |) in
    n1 + n2
```

Visualizing the task graph

parallel tuple expression

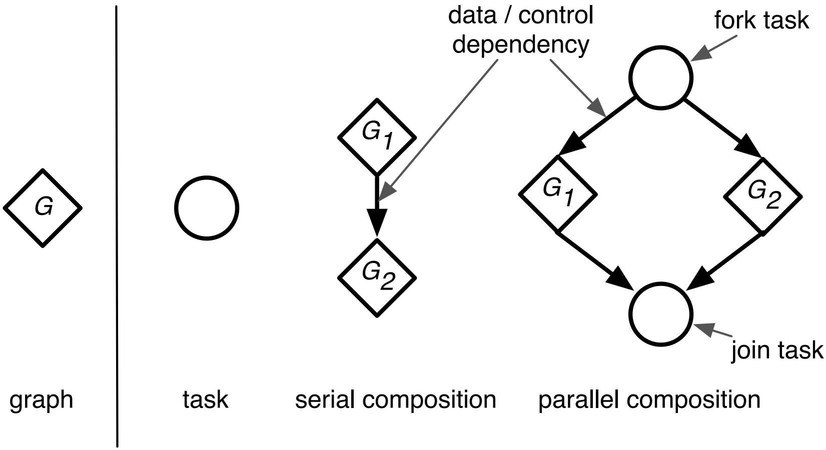
series-parallel DAG

```
sum (Node (Leaf 5,
     Node (Leaf 23,
           Leaf 8)))
```



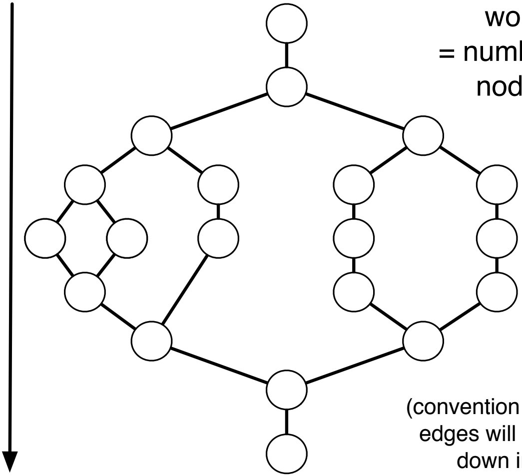
Series-Parallel DAGs

SP DAGs are built inductively from:



Work and span in SP DAGs

We use SP DAGs as basis for a cost model to estimate benefit of parallelism.

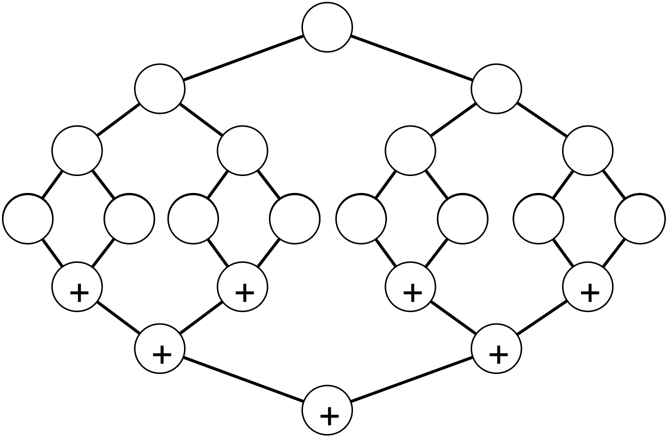


work
= number of
nodes

span
= length of
critical path

(convention: from hereon, edges will always point down implicitly)

The `sum` function applied to a balanced tree with 16 leaf nodes.



Work	Span
1	1
2	1
4	1
8	1
4	1
2	1
1	1
Total: 22	Total: 6

Defining costs

Let T_P denote the time to execute a given SP DAG with P processors.

- T_1 corresponds to the work.
- T_∞ corresponds to the span.
- Speedup with P processors is $\frac{T_1}{T_P}$.
  - Speedup P means “perfect linear”.
  - Speedup 1 means adding more processors does not help.

The average parallelism

The *average parallelism* P_{avg} is $\frac{T_1}{T_\infty}$, the ratio of work and span.

- Average parallelism represents the maximum speedup regardless of # processors.
  - Proof:
    - Speedup = $\frac{T_1}{T_P}$
    - $T_P \geq T_\infty$, for any P
    - Therefore, speedup $\leq \frac{T_1}{T_\infty}$
- We can use P_{avg} to
  - estimate how parallel a given algorithm is
  - compare parallelism of different algorithms
- Examples:
  - `sum` (balanced tree): large P_{avg}
  - `sum` (unbalanced tree): small P_{avg}
  - list-based mergesort: small P_{avg}
  - tree-based quicksort: large P_{avg}

Complexity of `sum`

- Suppose the input tree contains n leaves and has height h.
  - $T_1 = O(n)$
  - $T_\infty = O(h)$
- Example 1: the input tree is balanced ($h = \log_2 n$)
  - Large average parallelism $P_{avg} = O(\frac{n}{\log n})$
  - For instance, if $n = 2^{20}$, then $P_{avg} = \frac{2^{20}}{20} \approx 100,000$.
  - That is more than enough parallelism to utilize many processors.
- Example 2: the input tree is not balanced ($h = n$, *e.g.*, a list)
  - Small average parallelism $P_{avg} = 1$.
  - No benefit from having more than one processor.

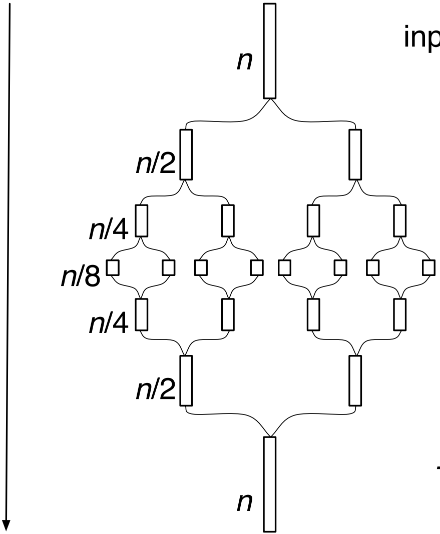
Merging two sorted lists

```
let rec merge (xs : int list, ys : int list) =
  match (xs, ys) with
    | ([], ys) -> ys
    | (xs, []) -> xs
    | (x::xs, y::ys) ->
      if x < y then
        x :: merge (xs, y::ys)
      else
        y :: merge (x::xs, ys)
```

List-based mergesort

```
let rec mergesort (xs : int list) =
  match xs with
    | [] -> []
    | [x] -> [x]
    | xs ->
      let med = length xs / 2 in
      let (left, right) =
            (take med xs, drop med xs) in
      merge (| mergesort left, mergesort right |)
```

Complexity of `mergesort`



input list length = n

$T_\infty = 2cn$

c is some constant

$T_1 = cn \log n$

Parallelism of `mergesort`

- Then the average parallelism $P_{avg} = \frac{cn \log n}{2cn} = \frac{\log n}{2}$.
- If $n = 2^{20}$, then $P_{avg} = \frac{\log 2^{20}}{2} = 10$.
- That is terrible: greatest speedup we can ever hope to achieve is 10x.
- Can we do better?
  - There exists a parallel functional mergesort with
    - $T_1 = O(n \log n)$
    - $T_\infty = O(\log^3 n)$
    - $P_{avg} = O(\frac{n}{\log^2 n})$
  - Basic ideas:
    - balanced trees (or arrays) instead of linked lists
    - parallelize the merging phase
  - See Blelloch & Greiner 1995 for more details.

Tree-based quicksort

```
type tree = Empty | Leaf of int | Node of tree * tree

let append (xs, ys) =
  match (xs, ys) with
   | (Empty, ys) -> ys
   | (xs, Empty) -> xs
   | _ -> Node (xs, ys)

let rec filter (f : int -> bool) (xs : tree) =
  match xs with
   | Empty -> Empty
   | Leaf x -> if f x then Leaf x else Empty
   | Node (xs, ys) ->
     let (xs', ys') =
       (| filter f xs, filter f ys |) in
     append (xs', ys')
```

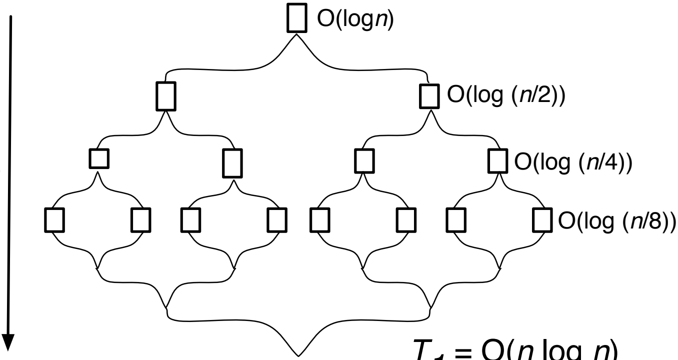
Tree-based quicksort

```
let rec quicksort (xs : tree) =
  match xs with
    | Empty -> Empty
    | Leaf x -> Leaf x
    | _ ->
      let pivot = first xs in
      let less    = filter (fun x -> x < pivot) xs in
      let greater = filter (fun x -> x > pivot) xs in
      let equal   = filter (fun x -> x = pivot) xs in
      let (left, right) =
        (| quicksort less, quicksort greater |) in
      append (left, append (equal, right))
```

Complexity of `quicksort` (assuming the input tree is balanced)

T_∞

$= O(\log^2 n)$



$T_1 = O(n \log n)$

$P_{avg} = O((n \log n) / \log n) = O(n / \log n)$ (good!)

Summary

	T_1	T_∞	P_{avg}
`sum` (balanced trees)	$O(n)$	$O(\log n)$	$O(\frac{n}{\log n})$
`sum` (unbalanced trees)	$O(n)$	$O(n)$	$O(1)$
`mergesort`	$O(n \log n)$	$O(n)$	$O(\log n)$
`quicksort`	$O(n \log n)$	$O(\log^2 n)$	$O(\frac{n}{\log n})$

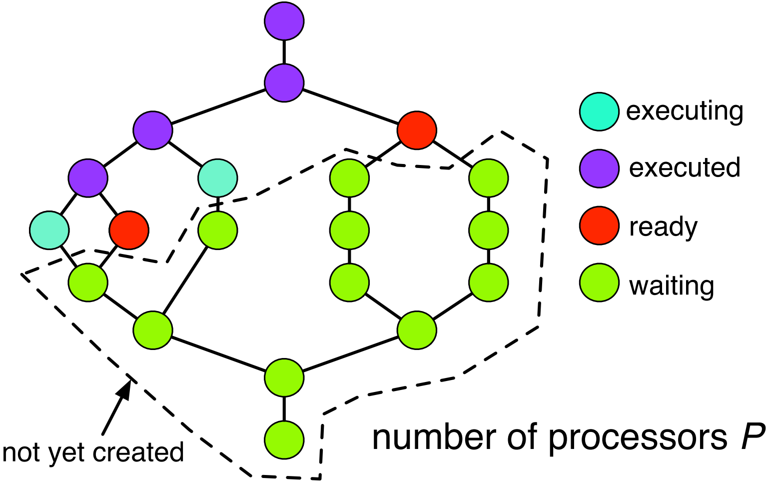
- Lists are bad.
- Trees are good.
- List-to-tree adaptation gives good results for a number of algorithms.
- Sometimes algorithms need to be redesigned.

Scheduling SP DAGs on multiprocessor machines

- Scheduling is mapping suparts of SP DAGs to finitely-many processors.
- Goal: to minimize execution time.
- Scheduler discovers the structure of the SP DAG as it goes.
  - *i.e.*, online scheduling
- The scheduling policy
  - determines order in which tasks are executed
  - mappings from tasks to processors

Greedy scheduling policies

A *greedy scheduler* is a scheduler in which no processor is idle if there are ready tasks.



number of processors $P = 2$

What's good about the greedy scheduler

Recall: T_P denotes execution time on P processors

- Observation 1: $T_P \geq T_\infty$
  - can go no faster than length of critical path
- Observation 2: $T_P \geq \frac{T_1}{P}$
  - can go no faster than having all processors always busy
- Brent's Theorem: $T_P \leq \frac{T_1}{P} + T_\infty$
- Theorem says that we can get close to optimal execution time (within factor of two).

When to expect linear speedups

Recall:

- $P_{avg} = \frac{T_1}{T_\infty}$.
- Brent's Theorem: $T_P \leq \frac{T_1}{P} + T_\infty$

Therefore:

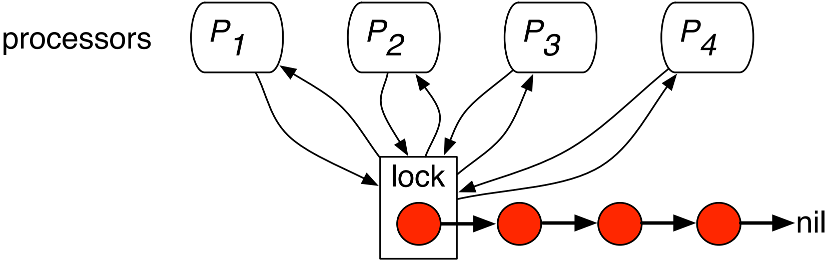
- Suppose that $P_{avg} \gg P \;\Leftrightarrow\; \frac{T_1}{T_\infty} \gg P \;\Leftrightarrow\; \frac{T_1}{P} \gg T_\infty$.
  - This case is often called "parallel slackness".
- With parallel slackness, the first term in Brent's Theorem dominates.
- So, we have $T_P \approx \frac{T_1}{P}$.
  - *i.e.*, linear speedup
- Observation: This prediction is valid for our model, where scheduling costs are not reflected.

Designing a greedy scheduling policy

First idea: maintain ready tasks in a shared queue.

- When a processor needs a new task, it grabs one from the queue.
- When a processor forks a task, it puts the task on the shared queue.

Problem with the shared queue

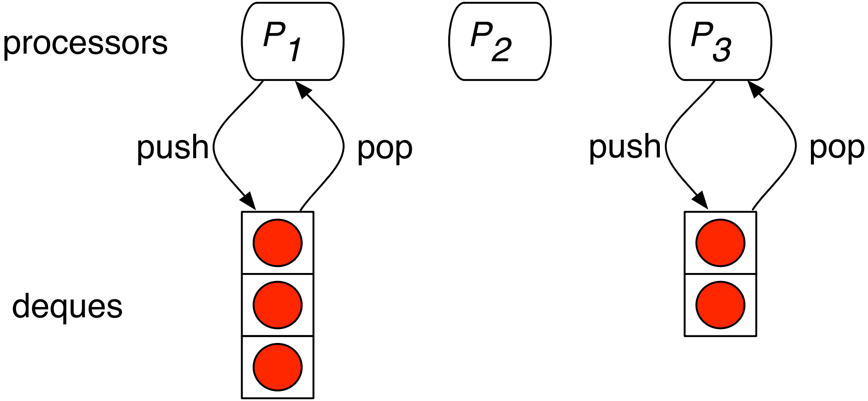


- Benefits of parallelism are obliterated because processors spend a lot of time waiting to access the queue.

Work stealing

- Each processor maintains the ready tasks that it has created in what is called a deque.
- Processors usually push and pop on their own deques.
- If a given processor's deque is empty, then the processor pops from the non-empty deque of another processor (if any).
  - called "stealing"
- There is an extensive literature on work stealing.
  - Burton and Sleep 1981; Halstead 1984; Mohr *et al.* 1990; Carlisle, *et al.* 1995; Leiserson, *et al.* 1995,1999; Arora *et al.* 1998; Acar *et al.* 2000; Danaher *et al.* 2005,2006; Agrawal and Leiserson 2008; Spoonhower 2010

Work stealing deques



processors

P_1

P_2

P_3

push

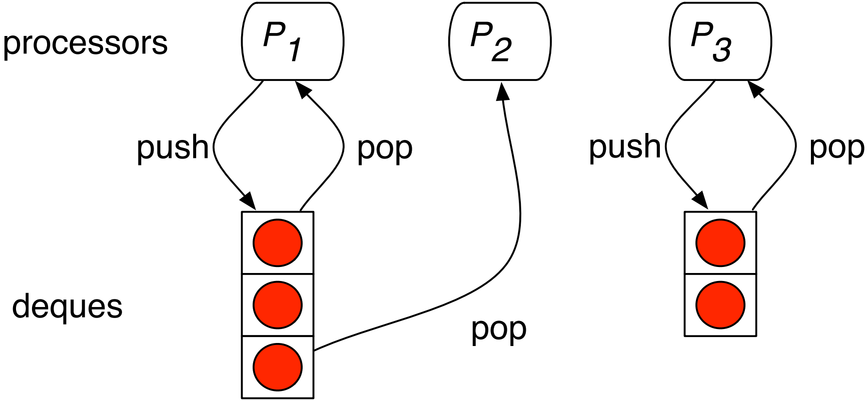
pop

push

pop

deques

Work stealing steal



processors

P_1

P_2

P_3

push

pop

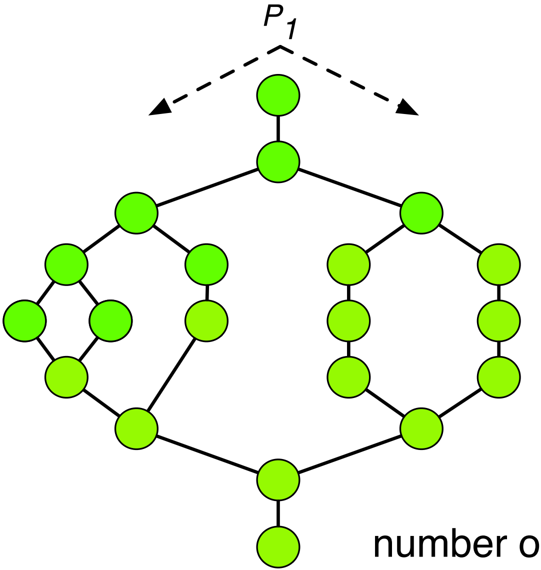
push

pop

deques

pop

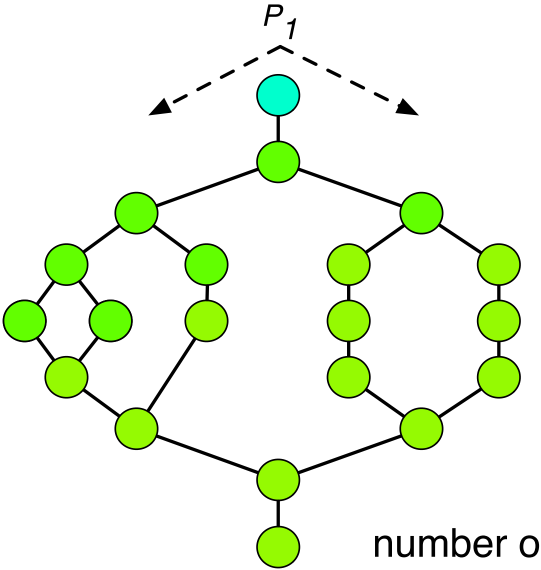
Example of work stealing



- 🟦 executing
- 🟪 executed
- 🟥 ready
- 🟩 waiting

number of processors $P = 2$

Example of work stealing



🔵 executing

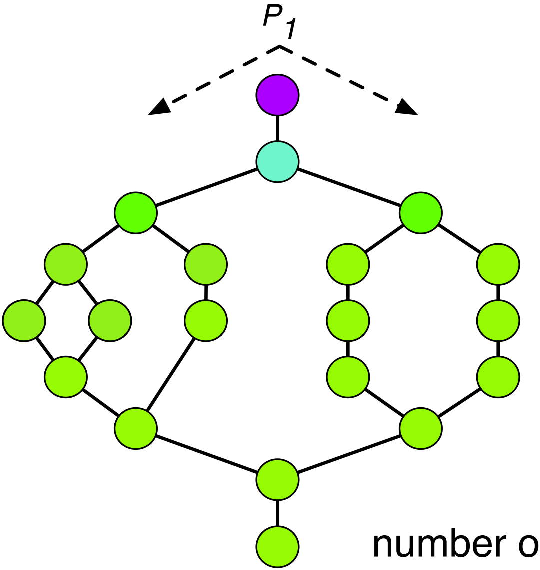
🟣 executed

🔴 ready

🟢 waiting

number of processors $P = 2$

Example of work stealing



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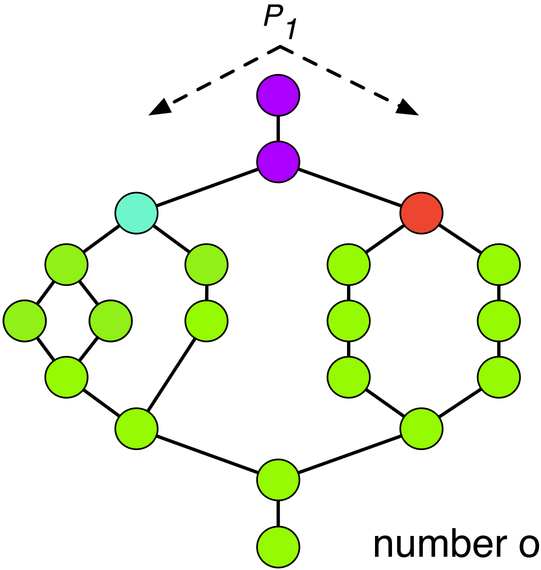
Example of work stealing



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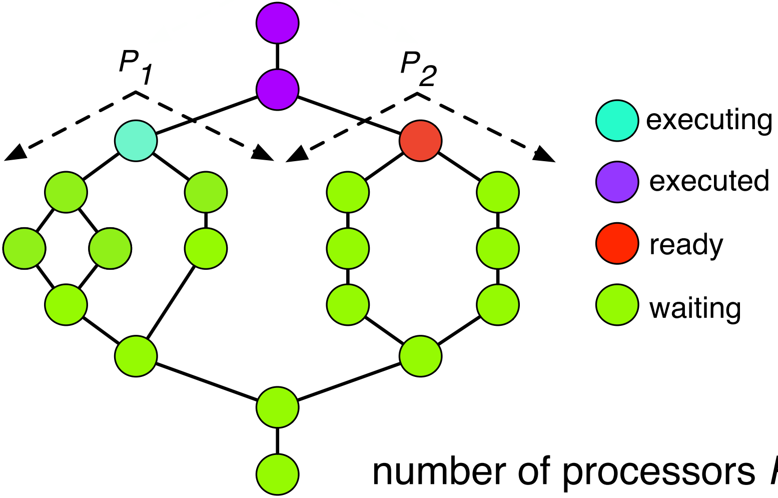
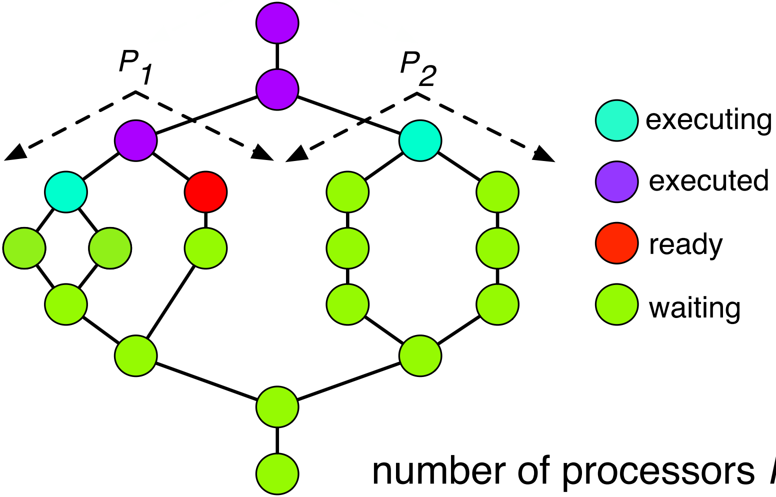
number of processors $P = 2$

Example of work stealing



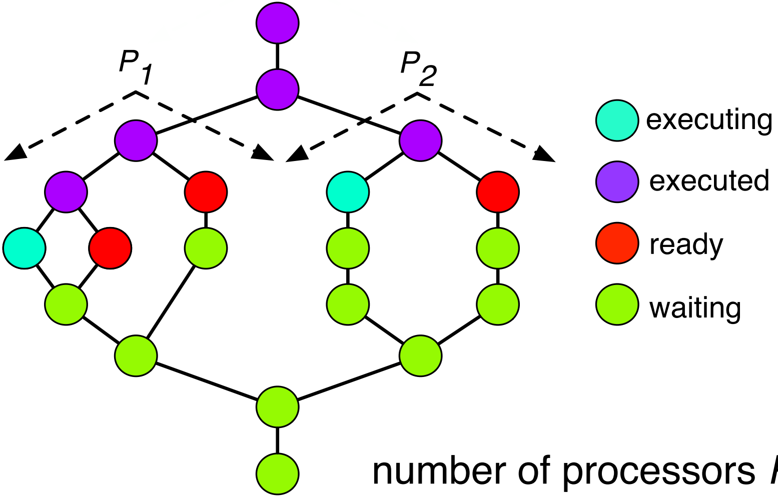
number of processors $P = 2$

Example of work stealing



number of processors $P = 2$

Example of work stealing



number of processors $P = 2$

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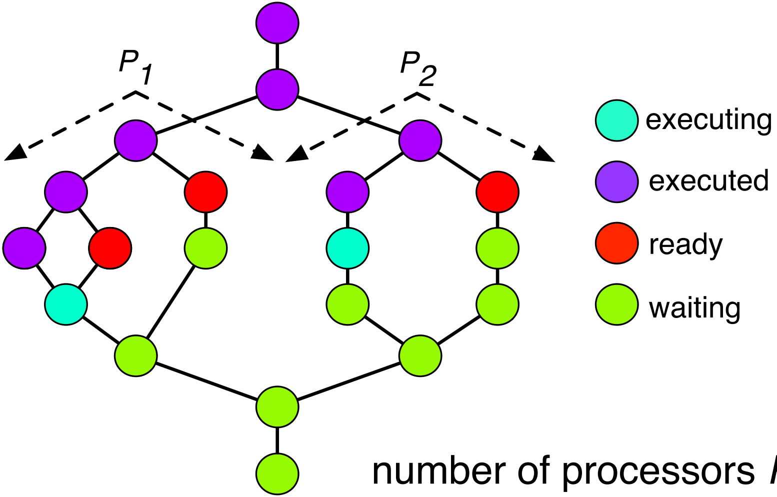
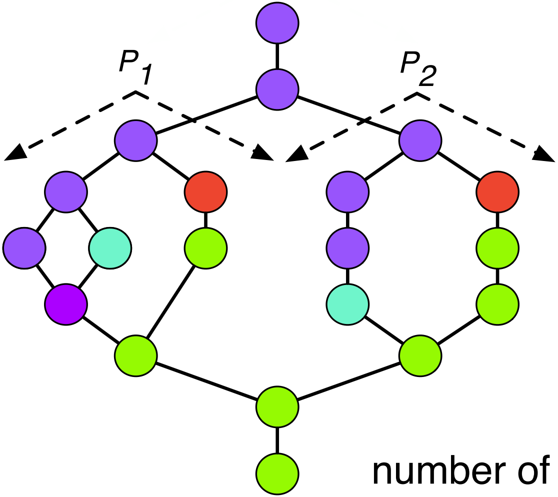


number of processors $P = 2$

Example of work stealing



- 🟦 executing
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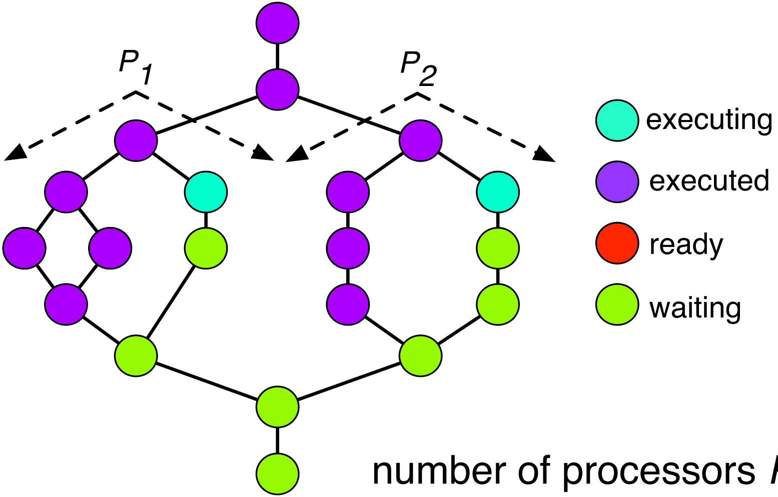
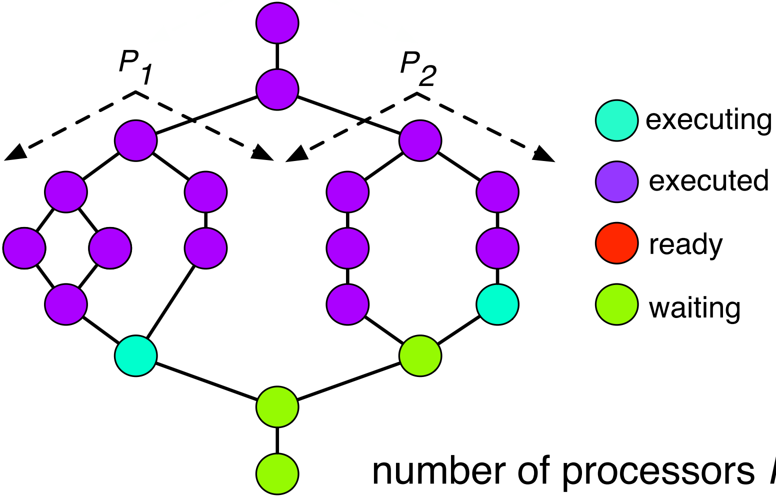
number of processors $P = 2$

Example of work stealing



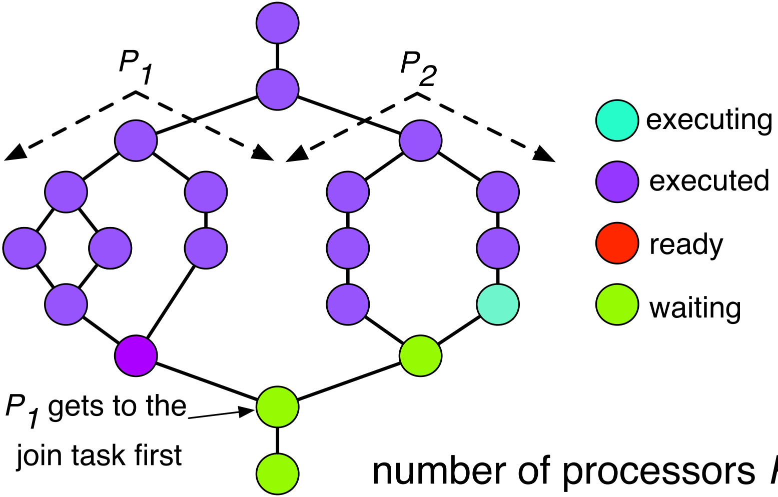
number of processors $P = 2$

Example of work stealing: handling join tasks



number of processors $P = 2$

Example of work stealing: handling join tasks

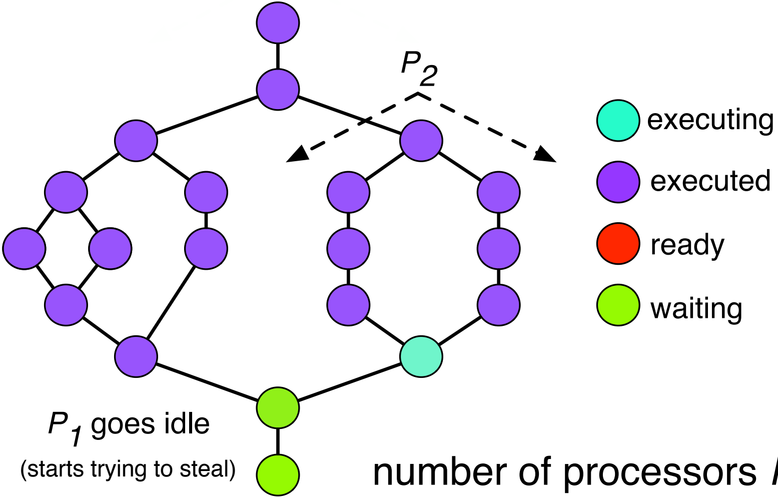


- 🟦 executing
- 🟪 executed
- 🟥 ready
- 🟩 waiting

P_1 gets to the join task first

number of processors $P = 2$

Example of work stealing: handling join tasks

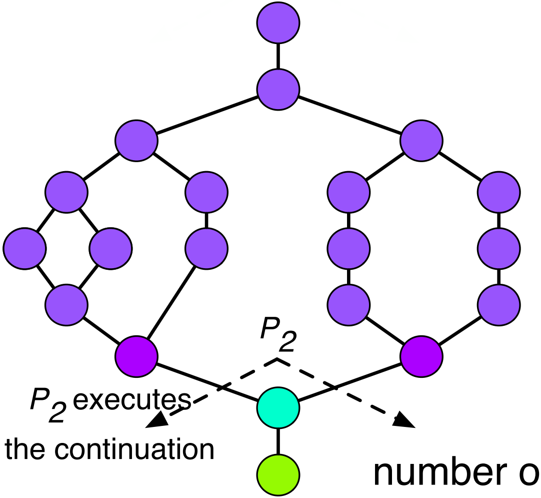


P_1 goes idle

(starts trying to steal)

number of processors $P = 2$

Example of work stealing: handling join tasks



🟦 executing

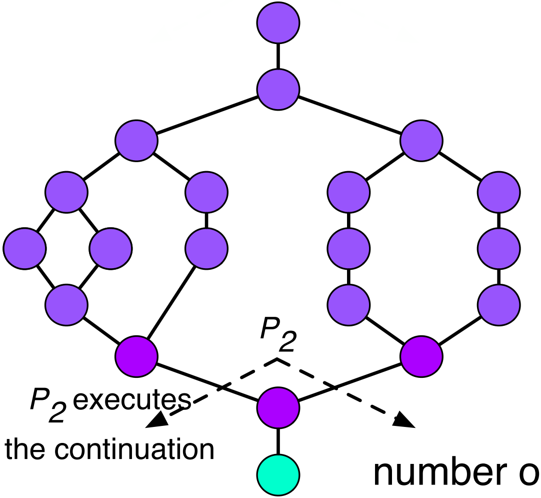
🟪 executed

🟥 ready

🟩 waiting

number of processors $P = 2$

Example of work stealing: handling join tasks



P_2 executes the continuation

🟦 executing

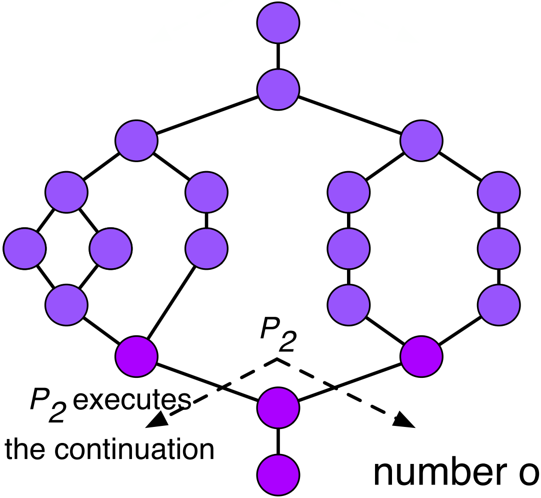
🟪 executed

🟥 ready

🟩 waiting

number of processors $P = 2$

Example of work stealing: handling join tasks



P_2 executes the continuation

🟦 executing

🟪 executed

🟥 ready

🟩 waiting

number of processors $P = 2$

The benefit of work stealing

Recall:

- Brent's theorem $T_P \leq \frac{T_1}{P} + T_\infty$
- When $\frac{T_1}{T_\infty} \gg P$, we can expect linear speedup: $T_P \approx \frac{T_1}{P}$.

But!

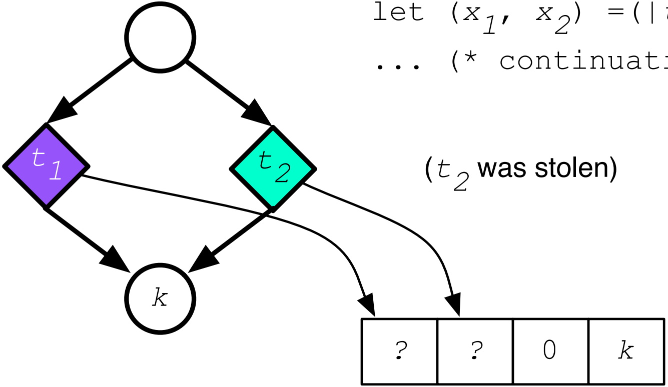
- We have to assume that ready tasks can be found efficiently.
- Work stealing achieves this because most of the time the ready task is in the local deque.
- Rarely does a processor have to steal.
  - Suppose we have the thief processor always pick its victim uniformly at random.
  - Blumofe and Leiserson (1995) show that, with high probability, expected total # of steals $\leq O(T_\infty P)$.
  - So, we want $T_\infty P \ll T_1$, which is equivalent to $\frac{T_1}{T_\infty} \gg P$.

The work-first principle for designing efficient implementations of work stealing

$$\begin{array}{ccccc} \text{scheduling costs} & = & \text{stealing costs} & + & \text{non stealing costs} \\ & & \text{(non local)} & & \text{(local)} \\ & & \text{(rare)} & & \text{(common)} \end{array}$$

Work-first principle (Frigo *et. al.* 1998): minimize the second term in the sum above because it represents the common case.

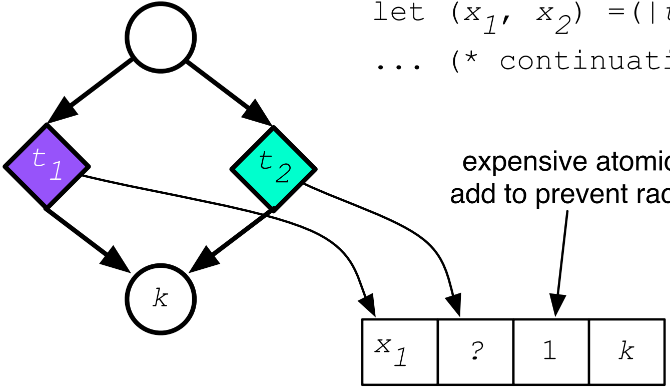
Slow clone (represents rare case)



let (x_1, x_2) =(| t_1, t_2 |) in
... (* continuation k *)

(t_2 was stolen)

Slow clone (represents rare case)



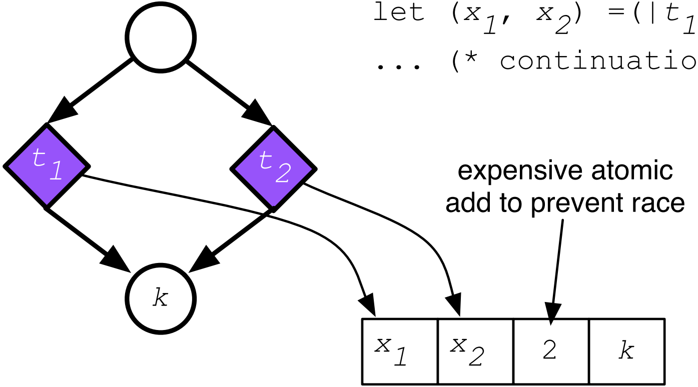
```
let (x₁, x₂) =(|t₁, t₂|) in
... (* continuation k *)
```

expensive atomic add to prevent race

Slow clone (represents rare case)

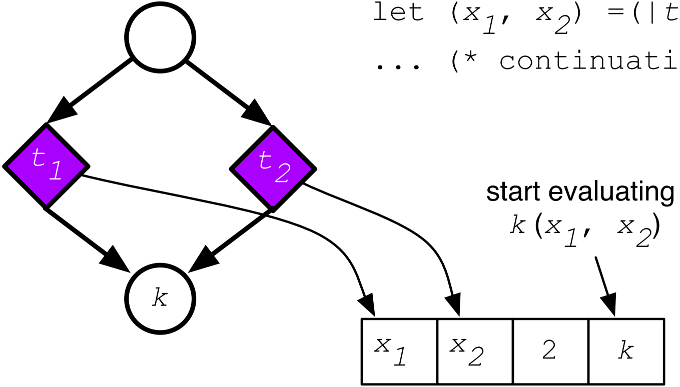
let (x_1, x_2) =(|t_1, t_2|) in

... (* continuation k *)



expensive atomic
add to prevent race

Slow clone (represents rare case)



let (x_1, x_2) =(|t_1, t_2|) in

... (* continuation k *)

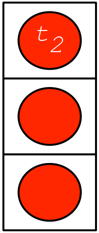
start evaluating

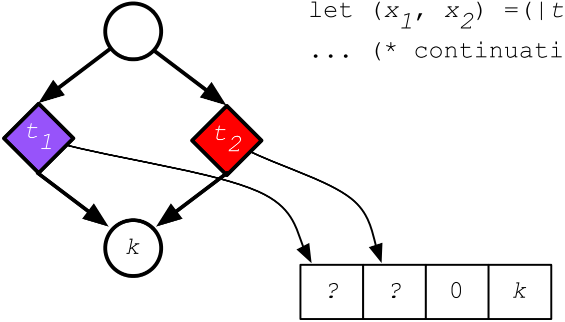
$k\,(x_1, x_2)$

Fast clone (represents common case)

let (x_1, x_2) =(|t_1, t_2|) in

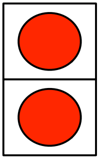
... (* continuation k *)





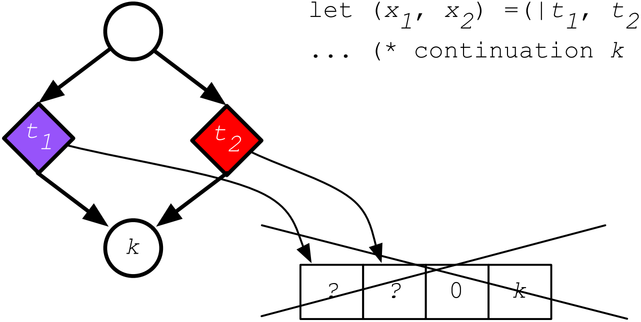
Fast clone (represents common case)

popped task t_2

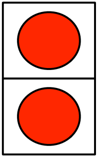


let (x_1, x_2) =(| t_1, t_2 |) in

... (* continuation k *)



Fast clone (represents common case)

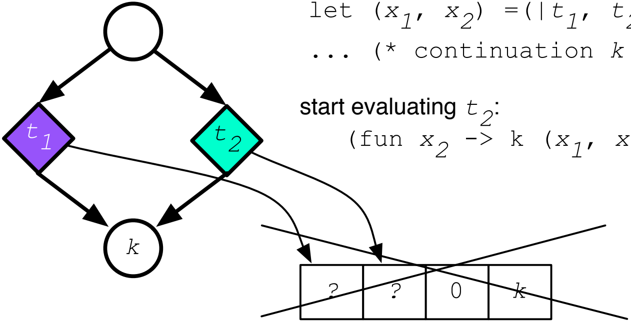


let (x_1, x_2) =(|t_1, t_2|) in

... (* continuation k *)

start evaluating t_2:

(fun x_2 -> k (x_1, x_2)) t_2



Making deques more efficient than using a lock

- There is a potential race in stealing because both thief and victim can try to pop from deque simultaneously.
- Using locks would be too expensive.
- There are some better approaches:
  - Private deques
  - Shared deques

Private deques

- Each processor has sole read / write access to its own deque.
  - Stealing is handled by message passing.
  - Designs investigated in Multilisp, ADM, and Manticore.
- Local deque access is cheap.
- Protecting deques from races is trivial: just delay handling a message while deque is in inconsistent state.
- Message-passing can be implemented on top of software polling or OS interrupts.
- In any case, busy processors always pay a cost for handling signals.
- We need to avoid the case where busy processors are sent too many messages.
- We can avoid sending unecessary messages by having each processor maintain a flag indicating if its deque is empty.

Shared deques

- Each processor has exclusive read / write access to the top of its own deque; all processors have read / write access to the bottom of the deque.
  - Synchronization is handled by Dijkstra-style mutual exclusion protocol (Frigo *et. al.* 1998).
  - Designs investigated in Multilisp, Cilk, and Hood.
- Non-blocking deques are crucial in the setting where processors are shared between work stealing and other processes.
  - Several papers investigate non-blocking deques.
    - Blumofe *et. al.* 1998
    - Nir & Shavit 2005
    - Tang *et. al.* 2010
- Presentations of concurrent deque algorithms assume sequential consistency.
  - Modern multicore machines usually have relaxed memory consistency models.
  - For such machines, expensive memory fences are required to prevent race condition.

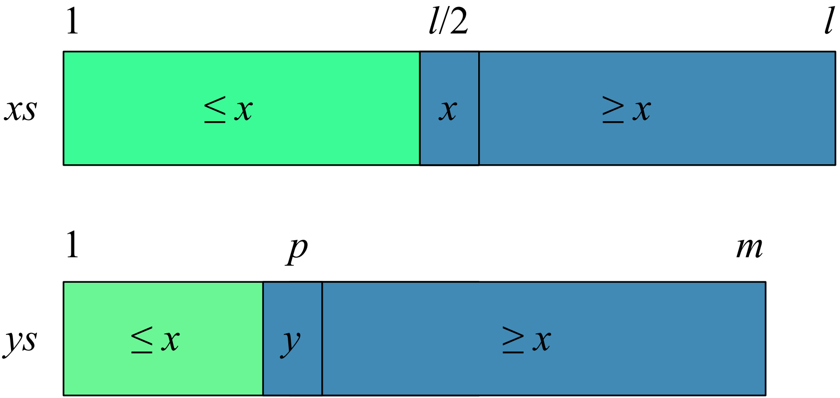
Comparing shared and private deques

- In private-deque approach, we can easily avoid costly memory fences, whereas we cannot in existing shared-deque approaches.
- Stealing is more expensive with with private deques because of message-passing overhead.
  - Stealing costs are arguably of minor importance, because we expect the common case is that # steals is negligable.
- Handling deque overflow is trivial with private deques; special concurrency protocol is necessary for shared deques (Nir & Shavit 2005).

Summary

- We introduced SP DAGs to model performance.
- We compared the parallelism of two tree-based and one list-based algorithms.
- We found that the tree-based algorithms naturally exhibit more parallelism than list-based ones.
- We studied the class of greedy schedulers and found that:
  - Execution time $T_P \leq \frac{T_1}{P} + T_\infty$
  - When $P_{avg} \gg P$, we achieve linear speedup.
- In practice, work stealing gets close to the bound above because it minimizes costs of managing ready tasks in common case.

Parallel `merge`



- Find p by binary search.
- *Key fact*: if the number of elements in both arrays is n, then the number of elements in the larger of the two recursive merges is no greater than $\frac{3}{4}n$.