Probabilistic Contracts for Component-based Design

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Probabilistic Contracts

System designers have to cope with multiple sources of uncertainty:

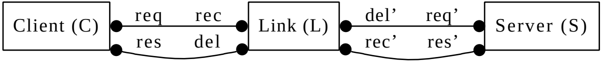
- Embedded and distributed systems usually encompass **unreliable** components.
- Behaviors of (black-box) components and the environment may be uncertain.
- **Abstraction** from complex deterministic behavior (“network access is available with p=95%”).

We want to describe properties such as:
“The probability that this component fails at this point of its behavior is $\leq 0.1\%$.”

We introduce **probabilistic contracts**, which distinguish **assumptions** on how a component is used from **guarantees** on the component behavior.

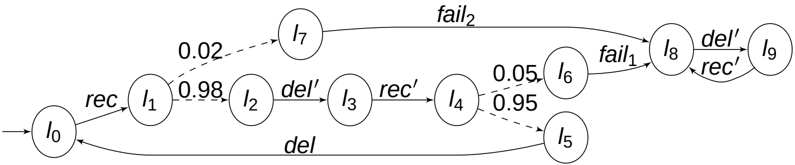
Interactive Markov Chain (IMC)

Example: client – link – server.



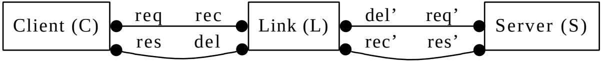
An IMC is an LTS with action states/transitions and probabilistic states/transitions [Hermanns 2002].

IMC used to model **component behaviors:**

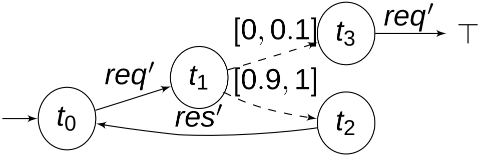


The IMC M_ℓ of the Link.

Probabilistic Contracts



A probabilistic contract is an IMC with probability intervals and a special $\top$ state:



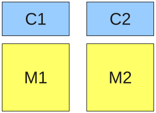
Contract C_s for Server

- action transitions leading to $\top$ are **assumed** not to be synchronized.
- action transitions not leading to $\top$ are **guaranteed** to be offered.
- actions not labelling any transition at a state are guaranteed not to be offered.

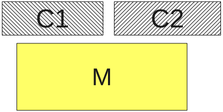
Operations for Contract-based Design Flow

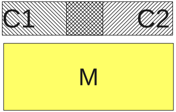
Essential operations:

- **refinement** and **satisfaction**;
- **parallel composition** ($C_1||_{\mathcal{I}}C_2$): E.g. $\mathcal{I} = \{a|d, b|e, c|f, g, u, v\}$



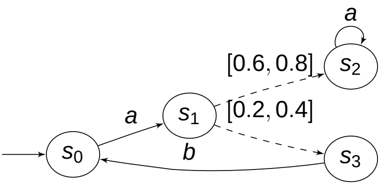
- **conjunction** of contracts ($C_1 \wedge C_2$):



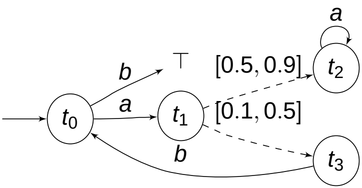


Additional definitions: **bisimulation, reduction, projection**

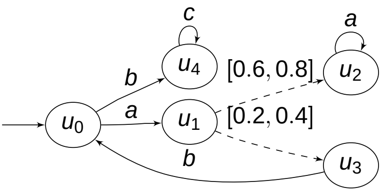
Contract Refinement



C_1



C_3

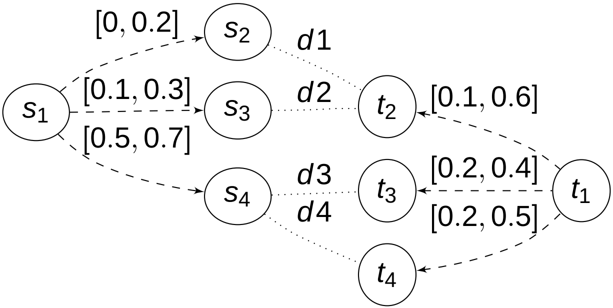


C_2

$C_1 \leq C_3$

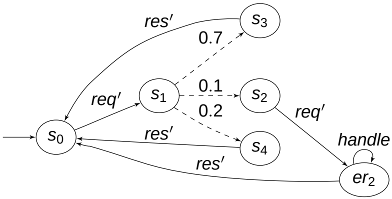
$C_2 \leq C_3$

Contract refinement for probabilistic states

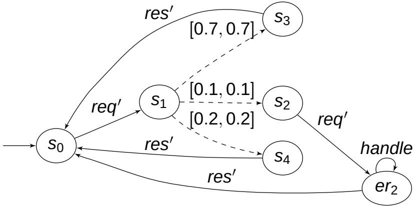


[Jonsson and Larsen : LICS'91]

Contract Satisfaction

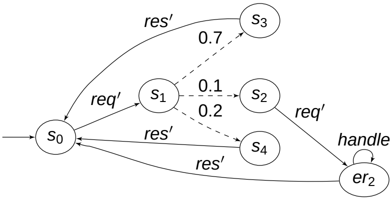


IMC M_s



Lifted IMC $\lfloor M_s \rfloor$

Contract Satisfaction



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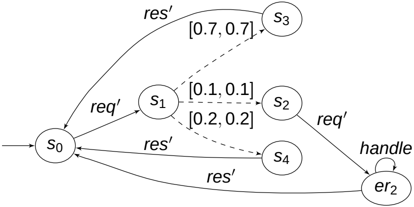
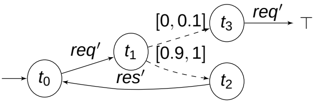


Lifted IMC $\lfloor M_s \rfloor$

Definition (Contract satisfaction)

An IMC M satisfies a contract C (written $M \models C$) iff $\lfloor M \rfloor \leq C$.



Contract C_s for Server

That is to check: $\boxed{s_0 \leq t_0}$

Contract Satisfaction

Definition (Models of contracts)

The set of models of a contract C (written $\mathcal{M}(C)$) is the set of IMCs that satisfy C: $\mathcal{M}(C) = \{M \mid M \models C\}$.

Definition (Semantical equivalence)

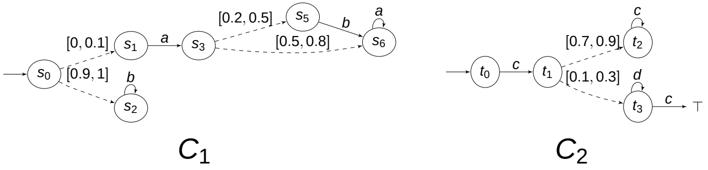
Contracts C_1 and C_2 are semantically equivalent (written $C_1 \equiv C_2$) iff $\mathcal{M}(C_1) = \mathcal{M}(C_2)$.

Lemma (Refinement and model inclusion)

For all contracts C_1 and C_2, if $C_1 \leq C_2$, then $\mathcal{M}(C_1) \subseteq \mathcal{M}(C_2)$.

Parallel Composition of contracts over two components

- A probabilistic transition has higher priority than an action transition.
- Interaction set $\mathcal{I}$: only transitions labeled with interactions in $\mathcal{I}$ can occur.
- Synchronize two probabilistic transitions.
- If one contract reaches $\top$, the composed contract reaches $\top$.



$C_1 || _{\mathcal{I}} C_2$ where $\mathcal{I}=\{a|c, b, d\}$

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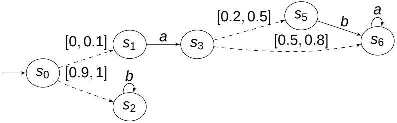
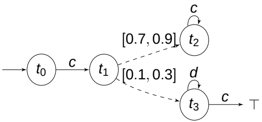
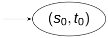


C_1



C_2



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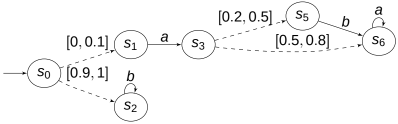

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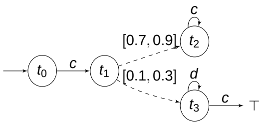
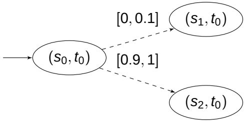


C_2



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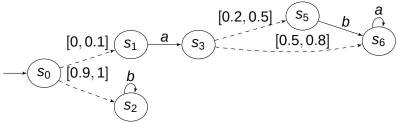

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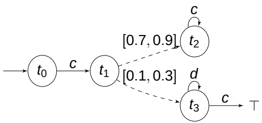
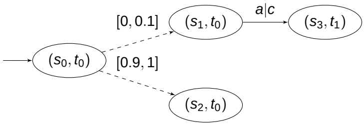


C_2



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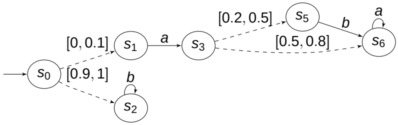

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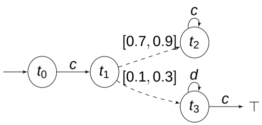
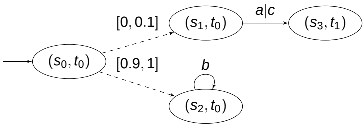


C_2



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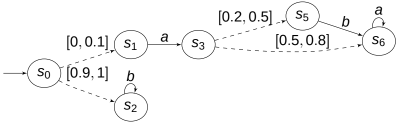

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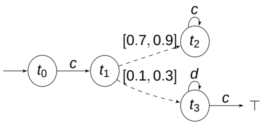
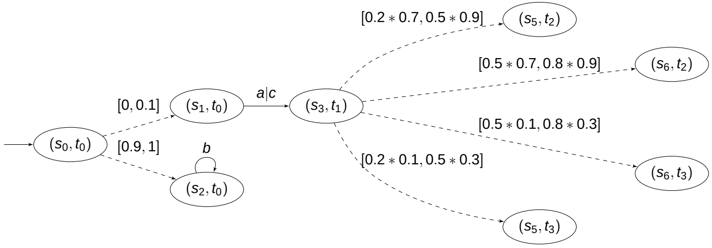


C_2



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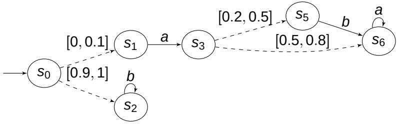

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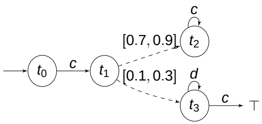
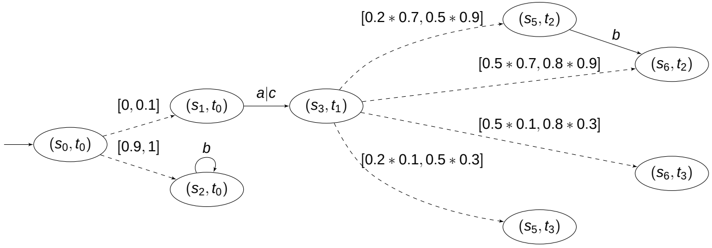


C_2



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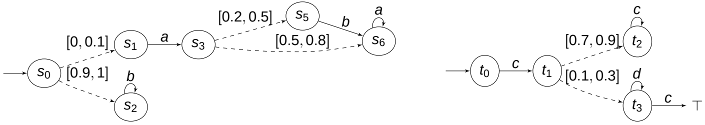
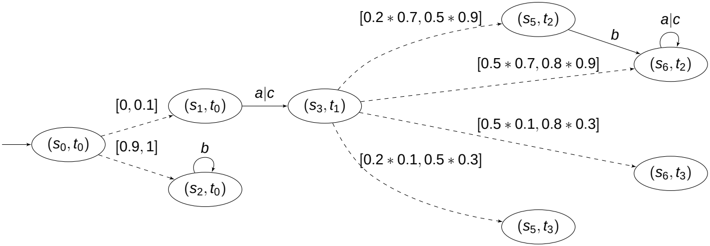


C_1

C_2



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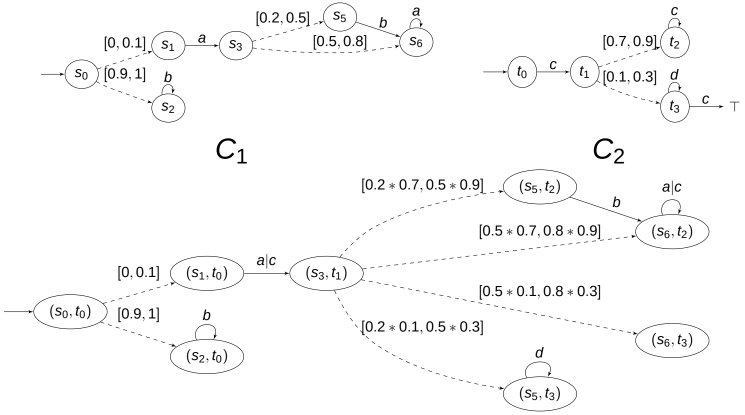

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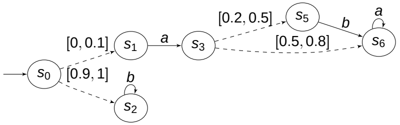

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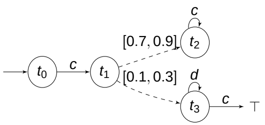
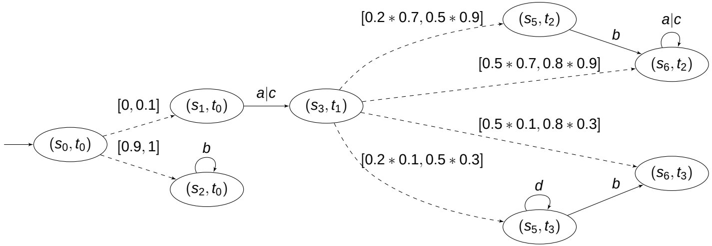


C_2



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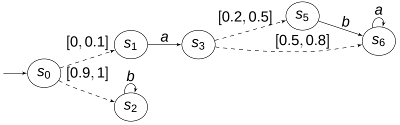

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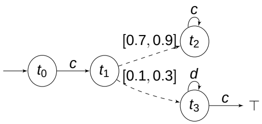
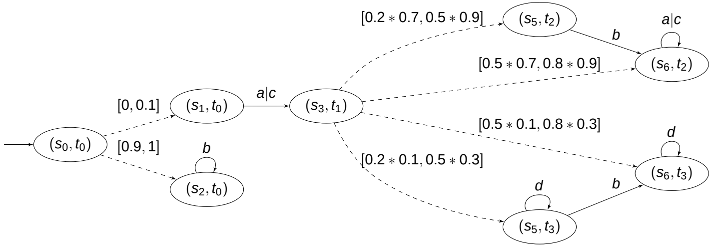


C_2



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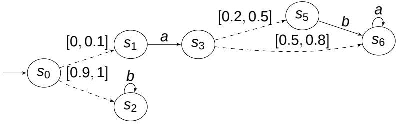

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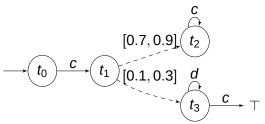
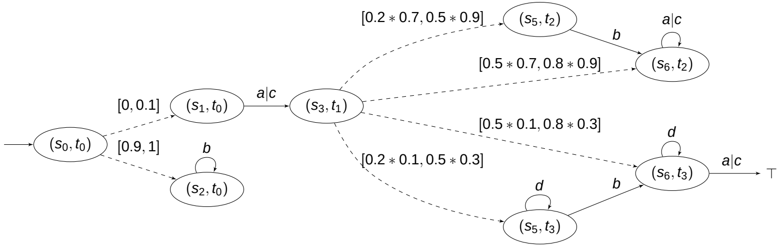


C_2



$C_1 || _{\mathcal{I}} C_2$ where $\mathcal{I}=\{a|c, b, d\}$

Properties for Parallel Composition

Theorem (Congruence of refinement for $||_{\mathcal{I}}$)

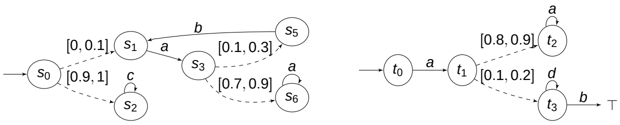
For all contracts C_1, C_2, C_3, C_4 and interaction set $\mathcal{I}$, if $C_1 \leq C_2$ and $C_3 \leq C_4$, then $C_1 ||_{\mathcal{I}} C_3 \ \leq \ C_2 ||_{\mathcal{I}} C_4$.

Theorem (Independent implementability)

For all IMCs M, N, contracts C_1, C_2, and interaction set $\mathcal{I}$, if $M \models C_1$ and $N \models C_2$, then $M ||_{\mathcal{I}} N \ \models \ C_1 ||_{\mathcal{I}} C_2$.

Conjunction: composition of requirements over a same component

- A probability transition has a higher priority than an action transition.
- Contracts must agree on common action transitions.
- Intersect probability intervals for two states that are similar.
- If one contract reaches $\top$, the conjunction behaves like the other contract.



C_1 with $A_1 = \{a, b, c\}$ $\qquad$ C_2 with $A_2 = \{a, b, d\}$

$$C_1 \wedge C_2$$

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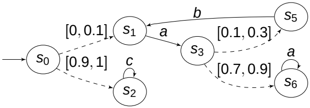
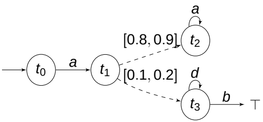
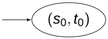


C_1 with $A_1 = \{a, b, c\}$



C_2 with $A_2 = \{a, b, d\}$



$C_1 \wedge C_2$

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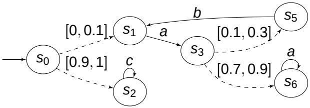
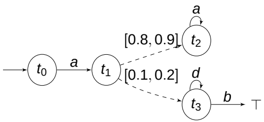
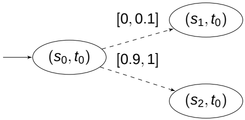


C_1 with $A_1 = \{a, b, c\}$



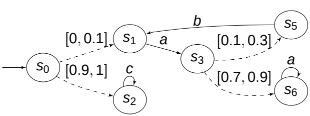
C_2 with $A_2 = \{a, b, d\}$



$C_1 \wedge C_2$

Conjunction: composition of requirements over a same component

- A probability transition has a higher priority than an action transition.
- Contracts must agree on common action transitions.
- Intersect probability intervals for two states that are similar.
- If one contract reaches $\top$, the conjunction behaves like the other contract.



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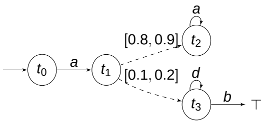

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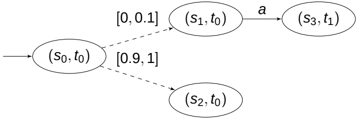

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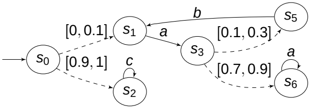

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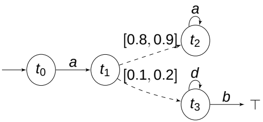

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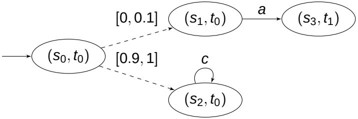

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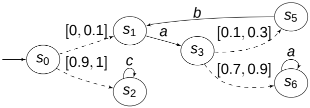

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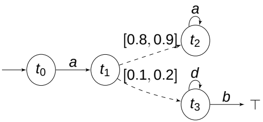

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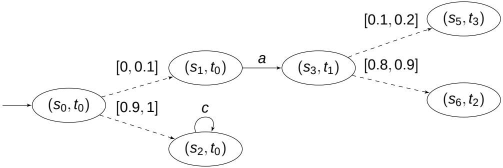

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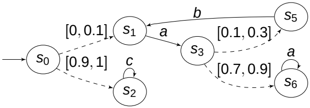

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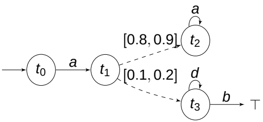

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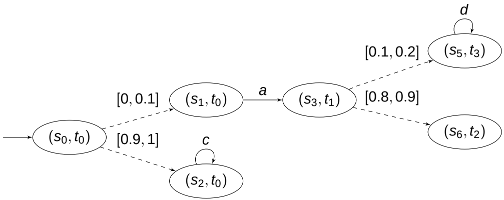

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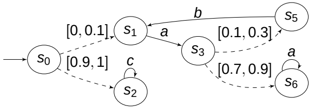

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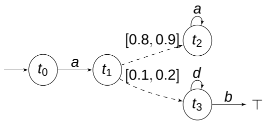

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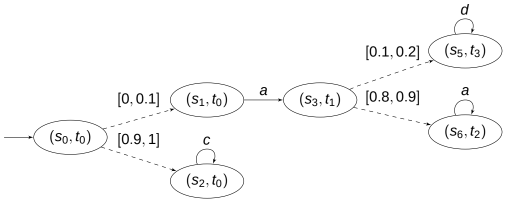

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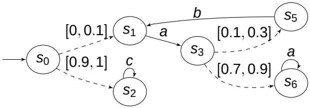

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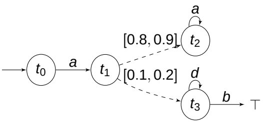

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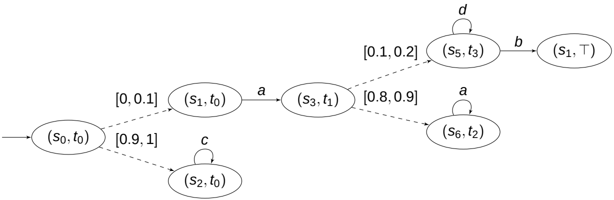

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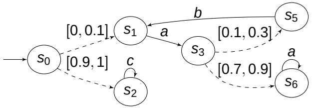

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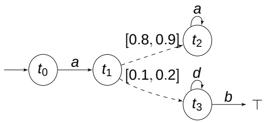

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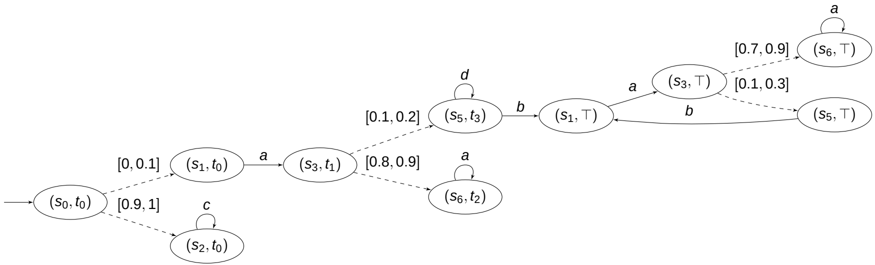
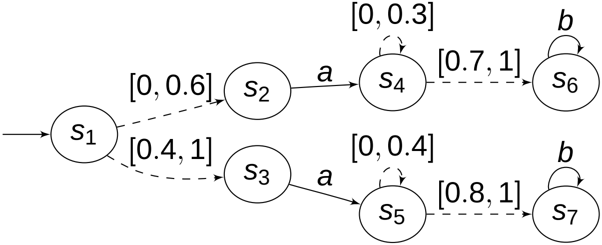


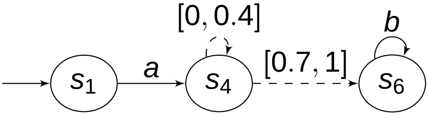
$C_1 \wedge C_2$

Unambiguous Contracts

For conjunction, we require the contracts to be **unambiguous**.



Ambiguous Contract



Unambiguous Contract

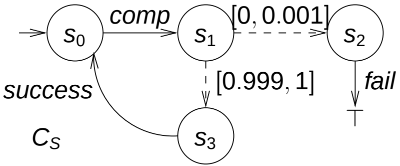
Properties of Conjunction

Theorem (Soundness of conjunction)

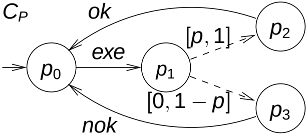
For all unambiguous contracts C_1 and C_2 with alphabets $\mathcal{A}$ such that:

$$C_1 \wedge C_2 \leq C_i \textit{ for } i = 1, 2$$

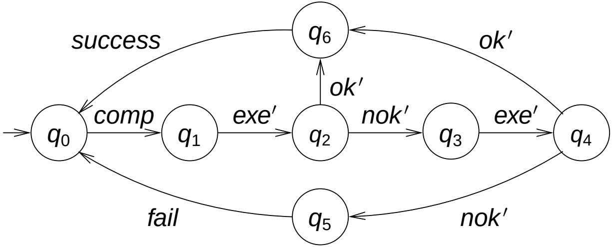
Case Study



Requirment C_S on the server



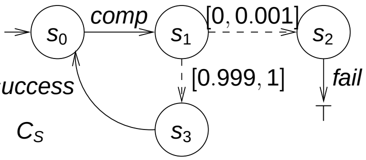
Contract C_P of a processor

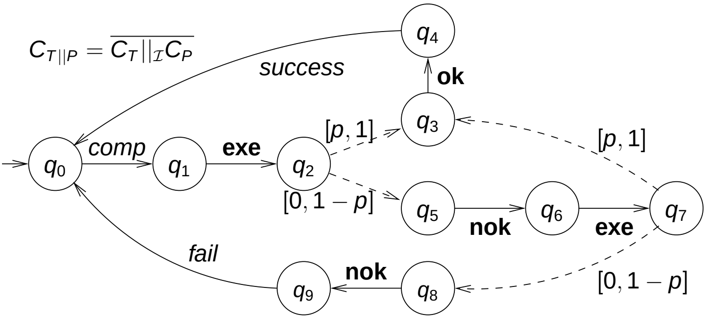


Contract C_T of a re-execution scheduler

$$\mathcal{I} = \{success, comp, fail, exe|exe', ok|ok', nok|nok'\}$$

Case Study

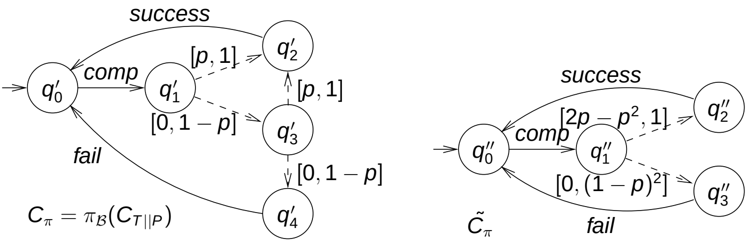




Shortcuts: **exe** = *exe*|*exe′*   **ok** = *ok*|*ok′*   **nok** = *nok*|*nok′*

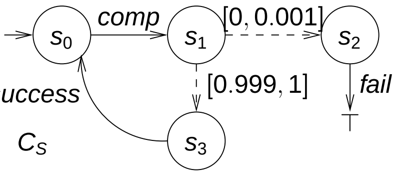
Case study: Refinement to Guarantee Reliability

- Collapse probabilistic transitions:



$\mathcal{B} = \{success, comp, fail\}$

- Refinement $\tilde{C}_\pi \leq C_S$ of reliability contract C_S gives constraint on p: $(1-p)^2 \leq 0.001$, that is, $p \geq 0.969$.



Conclusion

- Developed a probabilistic contract framework for component-based design.
- Provide operations for bottom-up and top-down design: *refinement*, *parallel composition*, and *conjunction*.
- Proved the desired properties of these operations.
- Small case study to show its usefulness.

Future work directions:

- Implement the framework in a tool, e.g. CADP model-checker
- Work on larger case studies.
- Study *blaming* (statically and at run-time).