Static Contract Checking for Haskell

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Program Errors Give Headache!

```
Module UserPgm where

f :: [Int] -> Int
f xs = head xs `max` 0

    :
    … f [] …
```

```
Module Prelude where

head :: [a] -> a
head (x:xs) = x
head [] = error "empty list"
```

Glasgow Haskell Compiler (GHC) gives at run-time

```
Exception: Prelude.head: empty list
```

From Types to Contracts

```
head (x:xs) = x

head :: [Int] -> Int
```

Type

... `(head 1)` ...

Bug!

```
not :: Bool -> Bool
not True = False
not False = True

null :: [a] -> Bool
null [] = True
null (x:xs) = False
```

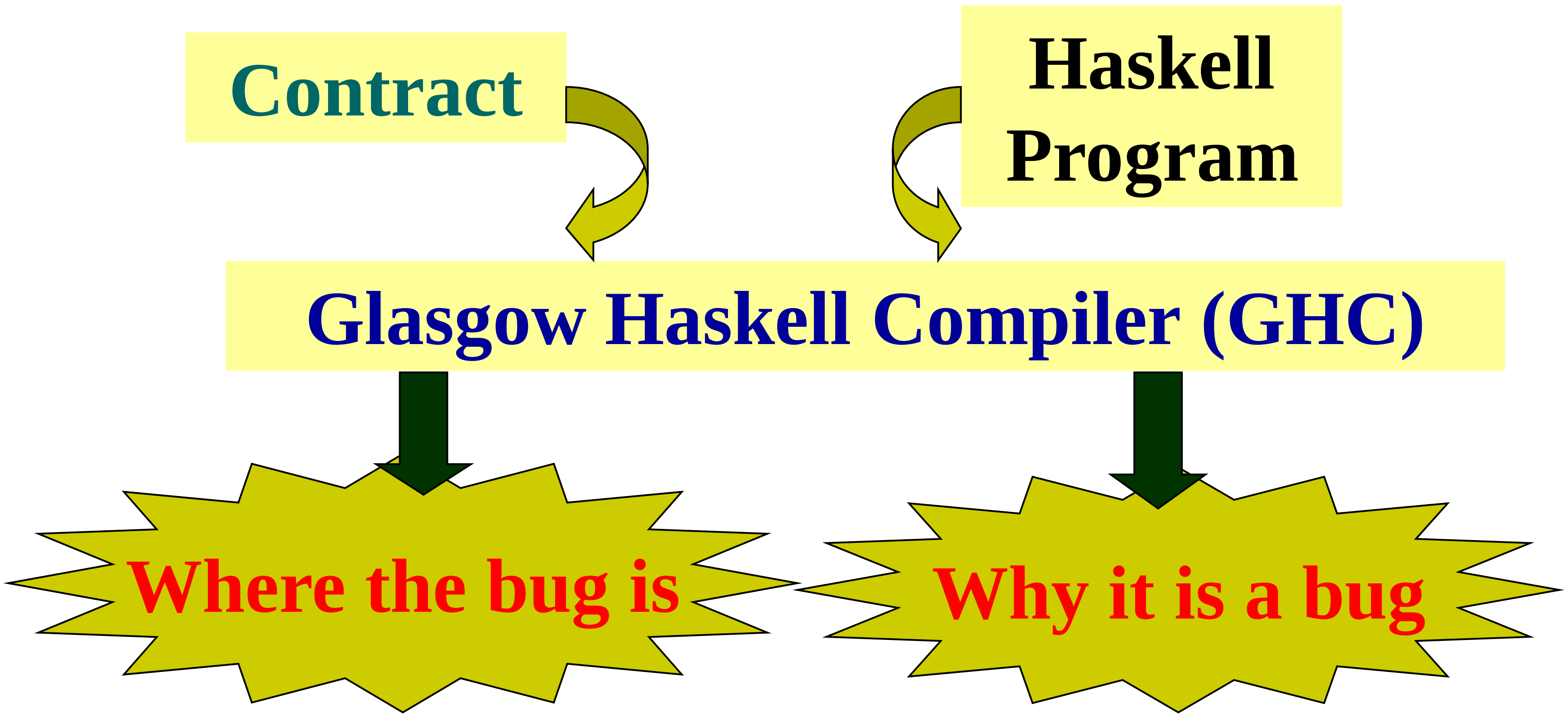
```
head ∈ {xs | not (null xs)} -> {r | True}
```

Contract
(original Haskell
boolean expression)

... `(head [])` ...

Bug!

What we want?


Contract
Haskell Program
Glasgow Haskell Compiler (GHC)
Where the bug is
Why it is a bug

Contract Checking

```
head ∈ {xs | not (null xs)} -> {r | True}
head (x:xs') = x

f xs = head xs `max` 0

Warning: f [] calls head
         which may fail head's precondition!

f_ok xs = if null xs then 0
          else head xs `max` 0
```

No more warnings from the compiler!

Satisfying a Predicate Contract

Arbitrary boolean-valued Haskell expression

$e \in \{x \mid p\}$ if (1) $p[e/x]$ gives True *and*

(2) e is crash-free.

`Recursive function, higher-order function, partial function can be called!`

Expressiveness of the Specification Language

```
data T = T1 Bool | T2 Int | T3 T T

sumT :: T -> Int
sumT ∈ {x | noT1 x} -> {r | True}
sumT (T2 a)      = a
sumT (T3 t1 t2) = sumT t1 + sumT t2

noT1 :: T -> Bool
noT1 (T1 _)      = False
noT1 (T2 _)      = True
noT1 (T3 t1 t2) = noT1 t1 && noT1 t2
```

Expressiveness of the Specification Language

```
sumT :: T -> Int
sumT ∈ {x | noT1 x} -> {r | True}
sumT (T2 a)      = a
sumT (T3 t1 t2) = sumT t1 + sumT t2

rmT1 :: T -> T
rmT1 ∈ {x | True} -> {r | noT1 r}
rmT1 (T1 a)      = if a then T2 1 else T2 0
rmT1 (T2 a)      = T2 a
rmT1 (T3 t1 t2) = T3 (rmT1 t1) (rmT1 t2)
```

For all crash-free `t::T`, `sumT (rmT1 t)` will not crash.

Higher Order Functions

```
all :: (a -> Bool) -> [a] -> Bool
all f [] = True
all f (x:xs) = f x && all f xs

filter :: (a -> Bool) -> [a] -> [a]
filter ∈ {f | True} -> {xs | True} -> {r | all f r}
filter f [] = []
filter f (x:xs’) = case (f x) of
                   True -> x : filter f xs’
                   False -> filter f xs’
```

Contracts for Higher-order Function's Parameter

```
f1 :: (Int -> Int) -> Int
f1 ∈ ({x | True} -> {y | y >= 0}) -> {r | r >= 0}
f1 g = (g 1) - 1

f2 :: {r | True}
f2 = f1 (\x -> x - 1)
```

```
Error: f1's postcondition fails
       when (g 1) >= 0 holds
            (g 1) - 1 >= 0 does not hold
Error: f2 calls f1
       which fails f1's precondition
```


[Findler&Felleisen:ICFP'02, Blume&McAllester:ICFP'04]

Various Examples

```
zip :: [a] -> [b] -> [(a,b)]
zip ∈ {xs | True} -> {ys | sameLen xs ys}
    -> {rs | sameLen rs xs }

sameLen [] []              = True
sameLen (x:xs) (y:ys) = sameLen xs ys
sameLen _ _                = False
```

```
f91 :: Int -> Int
f91 ∈ {n | True} -> {r | (n<=100 && r==91)
                         || r==n-10}
f91 n = case (n <= 100) of
          True -> f91 (f91 (n + 11))
          False -> n - 10
```

Sorting

```
(==>) True x = x
(==>) False x = True
```

```
sorted [] = True
sorted (x:[]) = True
sorted (x:y:xs) = x <= y && sorted (y : xs)

insert :: Int -> [Int] -> [Int]
insert ∈ {i | True} -> {xs | sorted xs} -> {r | sorted r}

merge :: [Int] -> [Int] -> [Int]
merge ∈ {xs | sorted xs}->{ys | sorted ys}->{r | sorted r}

bubbleHelper :: [Int] -> ([Int], Bool)
bubbleHelper ∈ {xs | True}
             -> {r | not (snd r) ==> sorted (fst r)}

insertsort, mergesort, bubblesort ∈ {xs | True}
                                 -> {r | sorted r}
```

AVL Tree

```
(&&) True x = x
(&&) False x = False
```

```
balanced :: AVL -> Bool
balanced L = True
balanced (N  t u) = balanced t && balanced u &&
                     abs (depth t - depth u) <= 1

data AVL = L | N Int AVL AVL
insert, delete :: AVL -> Int -> AVL
insert ∈ {x | balanced x} -> {y | True} ->
          {r | notLeaf r && balanced r     &&
               0 <= depth r - depth x      &&
                    depth r - depth x <= 1   }

delete ∈ {x | balanced x} -> {y | True} ->
          {r | balanced r && 0 <= depth x - depth r &&
                                depth x - depth r <= 1}
```

Functions without Contracts

```
data T = T1 Bool | T2 Int | T3 T T

noT1 :: T -> Bool
noT1 (T1 _)      = False
noT1 (T2 _)      = True
noT1 (T3 t1 t2) = noT1 t1 && noT1 t2

(&&) True x  = x
(&&) False x = False
```

No abstraction is more compact than the function definition itself!

Lots of Questions

- What does "crash" mean?
- What is "a contract"?
- What does it mean to "satisfy a contract"?
- How can we verify that a function does satisfy a contract?
- What if the contract itself diverges? Or crashes?

It's time to get precise...

What is the Language?

- Programmer sees Haskell
- Translated (by GHC) into Core language
  - Lambda calculus
  - Plus algebraic data types, and case expressions
  - BAD and UNR are (exceptional) values
  - Standard reduction semantics $e_1 \rightarrow e_2$

$$
\begin{array}{lll}
a, e, p & ::= & n \mid v \mid \lambda(x :: \tau).e \mid e_1\ e_2 \mid K\ \overrightarrow{e} \\
 & \mid & \mathtt{case}\ e_0\ \mathtt{of}\ alt_1 \ldots alt_n \mid \mathtt{BAD} \mid \mathtt{UNR} \\
alt & ::= & pt \rightarrow e \\
pt & ::= & K\ \overrightarrow{(x :: \tau)} \mid \mathtt{DEFAULT}
\end{array}
$$

Two Exceptional Values

Real Haskell Program

- **`BAD` is an expression that *crashes*.**

```
error :: String -> a
error s = BAD

head (x:xs) = x
head [] = BAD
```

```
div x y =
 case y == 0 of
 True -> error “divide by zero”
 False -> x / y

head (x:xs) = x
```

- **UNR (short for “unreachable”) is an expression that gets stuck. This is *not* a crash, although execution comes to a halt without delivering a result. (identifiable infinite loop)**

Crashing

Definition (Crash).

***A closed term e crashes* iff $e \rightarrow^* \mathrm{BAD}$**

Definition (Crash-free Expression)

***An expression e is crash-free* iff**

$\forall$ C. BAD $\notin_s$ C, $\vdash$ C[[e]] :: (), C[[e]] $\not\rightarrow^*$ BAD

Non-termination is not a crash

(i.e. partial correctness).

Crash-free Examples

	Crash-free?
(1,BAD)	NO
(1, True)	YES
`\x -> x`	YES
`\x -> if x > 0 then x else (BAD, x)`	NO
`\x -> if x*x >= 0 then x + 1 else BAD`	Hmm.. YES

Lemma: For all closed *e*,
***e* is syntactically safe $\Rightarrow$ *e* is crash-free.**

What is a Contract

(related to [Findler:ICFP02,Blume:ICFP04,Hinze:FLOPS06,Flanagan:POPL06])

$\mathtt{t} \in$ **Contract**		
`t ::=`	`{x	p}`	**Predicate Contract**
`|`	$\mathtt{x}:\mathtt{t}_1 \to \mathtt{t}_2$	**Dependent Function Contract**
`|`	$(\mathtt{t}_1, \mathtt{t}_2)$	**Tuple Contract**
`|`	`Any`	**Polymorphic Any Contract**

Full version: `x':{x | x >0} -> {r | r > x'}`

Short hand: `{x | x > 0} -> {r | r > x}`

`k:({x | x > 0} -> {y | y > 0}) -> {r | r > k 5}`

Questions on e ∈ t

```
3 ∈ {x | x > 0}
5 ∈ {x | True}

(True, 2)      ∈ {x | (snd x) > 0}  ?
(head [], 3)   ∈ {x | (snd x) > 0} ?
BAD            ∈ ?
  ?            ∈ {x | False}
  ?            ∈ {x | BAD}
\x-> BAD       ∈ {x | False} -> {r | True} ?
\x-> BAD       ∈ {x | True} ->  ?
\x-> x         ∈ {x | True} ?
```

What exactly does it mean to say that

e “satisfies” contract t?

$$e \in t$$

Contract Satisfaction

(related to [Findler:ICFP02,Blume:ICFP04,Hinze:FLOPS06])

Given $\vdash e :: \tau$ and $\vdash_c t :: \tau$, we define $e \in t$ as follows:

$e \in \{x \mid p\} \quad \Leftrightarrow \quad e\uparrow$ or (e is crash-free and $p[e/x] \not\rightarrow^* \{BAD, False\}$) **[A1]**

$e \in x{:}t_1 \rightarrow t_2 \quad \Leftrightarrow \quad e\uparrow$ or ($e \rightarrow^* \lambda x.e'$ and $\forall e_1 \in t_1.\ (e\ e_1) \in t_2[e_1/x]$) **[A2]**

$e \in (t_1, t_2) \quad \Leftrightarrow \quad e\uparrow$ or ($e \rightarrow^* (e_1,e_2)$ and $e_1 \in t_1$ and $e_2 \in t_2$) **[A3]**

$e \in$ `Any` $\quad \Leftrightarrow \quad$ True **[A4]**

$e\uparrow$ means e diverges or $e \rightarrow^*$ `UNR`

Only Crash-free Expression Satisfies a Predicate Contract

$e \in \{x \mid p\}$	$\Leftrightarrow$	$e\uparrow$ or (e is crash-free and $p[e/x] \not\rightarrow^{*} \{BAD, False\}$
$e \in x{:}t_1 \rightarrow t_2$	$\Leftrightarrow$	$e\uparrow$ or ($e \rightarrow^{*} \lambda x.e'$ and $\forall e_1 \in t_1.\ (e\ e_1) \in t_2[e_1/x]$)
$e \in (t_1, t_2)$	$\Leftrightarrow$	$e\uparrow$ or ($e \rightarrow^{*} (e_1,e_2)$ and $e_1 \in t_1$ and $e_2 \in t_2$)
$e \in$ `Any`	$\Leftrightarrow$	True

	YES or NO?
`(True, 2)` $\in$ `{x	(snd x) > 0}`	YES
`(head [], 3)` $\in$ `{x	(snd x) > 0}`	NO
`\x-> x` $\in$ `{x	True}`	YES
`\x-> x` $\in$ `{x	loop}`	YES
`5` $\in$ `{x	BAD}`	NO
`loop` $\in$ `{x	False}`	YES
`loop` $\in$ `{x	BAD}`	YES

All Expressions Satisfy `Any`

```
fst ∈ ({x | True}, Any) -> {r | True}
fst (a,b) = a

g x = fst (x, BAD)
```

Inlining may help, but breaks down when function definition is big or recursive

	YES or NO?
`5 ∈ Any`	YES
`BAD ∈ Any`	YES
`(head [], 3) ∈ (Any, {x | x> 0})`	YES
`\x -> x ∈ Any`	YES
`BAD ∈ Any -> Any`	NO
`BAD ∈ (Any, Any)`	NO

All Contracts are Inhabited

$e \in \{x \mid p\}$	$\Leftrightarrow$	$e\uparrow$ or (e is crash-free and $p[e/x] \not\to^* \{BAD, False\}$
$e \in x{:}t_1 \to t_2$	$\Leftrightarrow$	$e\uparrow$ or ($e \to^* \lambda x.e'$ and $\forall\, e_1 \in t_1.\ (e\ e_1) \in t_2[e_1/x]$)
$e \in (t_1, t_2)$	$\Leftrightarrow$	$e\uparrow$ or ($e \to^* (e_1,e_2)$ and $e_1 \in t_1$ and $e_2 \in t_2$)
$e \in$ `Any`	$\Leftrightarrow$	True

	YES or NO?
`\x-> BAD ∈ Any -> Any`	YES
`\x-> BAD ∈ {x	True} -> Any`	YES
`\x-> BAD ∈ {x	False} -> {r	True}`	NO

Blume&McAllester[JFP'06] say YES

What to Check?

Does function *f* satisfy its contract *t* (written $f \in t$)?

At the definition of each function *f*,

Check $f \in t$ assuming all functions called in *f* satisfy their contracts.

Goal: `main` $\in$ {x | True}

(`main` is *crash-free*, hence the program cannot crash)

How to Check?

Part I

Define
$e \in t$

Grand Theorem
$e \in t \quad \Leftrightarrow \quad e \triangleright t$ is crash-free

(related to Blume&McAllester:JFP'06)

Construct
$e \triangleright t$
(e "ensures" t)

Part II

Simplify $(e \triangleright t)$

some e'

If e' is syntactically safe,
then **Done!**

What we can’t do?

```
g1, g2 ∈ Ok -> Ok
g1 x = case (prime x > square x) of
          True -> x
          False -> error "urk"
```

Crash!

```
g2 xs ys =
  case (rev (xs ++ ys) == rev ys ++ rev xs) of
    True -> xs
    False -> error "urk"
```

Crash!

Hence, three possible outcomes:

(1) Definitely Safe (no crash, but may loop)

(2) Definite Bug (definitely crashes)

(3) Possible Bug

Wrappers $\triangleright$ and $\triangleleft$
($\triangleright$ pronounced ensures $\triangleleft$ pronounced requires)

```
e ▷ {x | p} = case p[e/x] of
                 True -> e
                 False -> BAD
```

$$e \triangleright x{:}t_1 \to t_2$$
$$= \lambda\, v.\; (e\; (v \triangleleft t_1)) \triangleright t_2[v \triangleleft t_1 / x]$$

$$e \triangleright (t_1,\; t_2) = \texttt{case } e \texttt{ of}$$
$$(e_1,\; e_2) \texttt{ -> } (e_1 \triangleright t_1,\; e_2 \triangleright t_2)$$

```
e ▷ Any = UNR
```

related to [Findler:ICFP02,Blume:JFP06,Hinze:FLOPS06]

Wrappers $\rhd$ and $\lhd$
($\rhd$ pronounced ensures $\lhd$ pronounced requires)

```
e ◁ {x | p} = case p[e/x] of
                True -> e
                False -> UNR
```

$e \lhd x{:}t_1 \rightarrow t_2$
$\quad = \lambda v.\ (e\ (v \rhd t_1)) \lhd t_2[v \rhd t_1/x]$

$e \lhd (t_1, t_2) = \mathtt{case}\ e\ \mathtt{of}$
$\qquad (e_1, e_2) \to (e_1 \lhd t_1, e_2 \lhd t_2)$

$e \lhd \mathtt{Any} = \mathtt{BAD}$

related to [Findler:ICFP02,Blume:JFP06,Hinze:FLOPS06]

Example

```
head:: [a] -> a
head []      = BAD
head (x:xs)  = x
```

head ∈ { xs | not (null xs) } -> Ok

head ▷{xs | not (null xs)} -> Ok

= \v. head (v ◁ {xs | not (null xs)}) ▷ Ok

e ▷ *Ok* = *e*

= \v. head (v ◁ {xs | not (null xs)})

```
= \v. head (case not (null v) of
                True -> v
                False -> UNR)
```

```
\v. head (case not (null v) of
               True -> v
               False -> UNR)
```

```
null :: [a] -> Bool
null []      = True
null (x:xs)  = False

not :: Bool -> Bool
not True  = False
not False = True
```

Now inline 'not' and 'null'

```
= \v. head (case v of
               [] -> UNR
               (p:ps) -> p)
```

Now inline 'head'

```
= \v. case v of
        []      -> UNR
        (p:ps)  -> p
```

So head [] fails with UNR, not BAD, blaming the caller

Higher-Order Function

```
f1 :: (Int -> Int) -> Int
f1 ∈ ({x | True} -> {y | y >= 0}) -> {r | r >= 0}
f1 g = (g 1) - 1

f2:: {r | True}
f2 = f1 (\x -> x - 1)
```

```
f1 ▷ ({x | True} -> {y | y >= 0}) -> {r | r >= 0}
= …   ▷               ◁                  ▷
= λ v1. case (v1 1) >= 0 of
        True -> case (v1 1) - 1 >= 0 of
                True -> (v1 1) -1
                False -> BAD
        False -> UNR
```

Grand Theorem
$e \in t \iff e \rhd t$ is crash-free

```
e ▷ {x | p} = case p[e/x] of
                  True -> e
                  False -> BAD
```

```
loop ∈{x | False}
loop ▷{x | False}
= case False of {True -> loop; False -> BAD}
= BAD,
```
which is not crash-free

```
BAD ∉ Ok -> Any
BAD ▷ Ok -> Any
= \v -> ((BAD (v ◁ Ok)) ▷ Any
= \v -> UNR,
```
which is crash-free

☹

Grand Theorem
e ∈ t ⇔ e ▷ t is crash-free

```
e ▷ {x | p} = e `seq` case p[e/x] of
                      True -> e
                      False -> BAD
```

```
e_1 `seq` e_2 = case e_1 of {DEFAULT -> e_2}
```

```
loop ∈{x | False}
loop ▷{x | False}
= loop `seq` case False of {…}
= loop,
```
which is crash-free

```
BAD ∉ Ok -> Any
BAD ▷ Ok -> Any
= BAD `seq` \v -> ((BAD (v ◁ Ok)) ▷ Any
= BAD,
```
which is not crash-free

☺

Contracts that Diverge

`\x->BAD ∈ {x | loop} ?` NO

But

`\x->BAD ▷ {x | loop}`

```
= \x->BAD `seq` case loop of
                    True -> \x -> BAD
                    False -> BAD
```

crash-free

```
e ▷ {x | p} = e `seq` case fin p[e/x] of
                          True -> e
                          False -> BAD
```

`fin` converts divergence to `True`

Contracts that Crash

Grand Theorem
$e \in t \quad \Leftrightarrow \quad e \triangleright t$ is crash-free

- … much trickier
  - ($\Rightarrow$) does not hold, ($\Leftarrow$) still holds
- Open Problem
  - Suppose `fin` converts `BAD` to `False`
  - Not sure if Grand Theorem holds because NO proof, and NO counter example either.

Well-formed Contracts

Grand Theorem

$e \in t \quad \Leftrightarrow \quad e \triangleright t$ is crash-free

Well-formed t

t is Well-formed (WF) iff

t = {x | p} and p is crash-free

or t = $x{:}t_1 \rightarrow t_2$ and t_1 is WF and $\forall e_1 \in t_1$, $t_2[e_1/x]$ is WF

or t = (t_1, t_2) and both t_1 and t_2 are WF

or t = Any

Properties of $\rhd$ and $\lhd$

Key Lemma:
For all closed, crash-free *e*, and closed *t*,
$(e \lhd t) \in t$

Projections: (related to Findler&Blume:FLOPS'06)
For all e and t, if e $\in$ t, then

(a) $e \preceq e \rhd t$

(b) $e \lhd t \preceq e$

Definition (Crashes-More-Often):
$e_1 \preceq e_2$ iff for all C, $\vdash C[[e_i]] :: ()$ for i=1,2 and
$C[[e_2]] \to^* BAD \Rightarrow C[[e_1]] \to^* BAD$

More Lemmas ☺

Lemma [Monotonicity of Satisfaction]:

If $e_1 \in t$ and $e_1 \preceq e_2$, then $e_2 \in t$

Lemma [Congruence of $\preceq$]:

$e_1 \preceq e_2 \quad \Rightarrow \quad \forall C.\ C[[e_1]] \preceq C[[e_2]]$

Lemma [Idempotence of Projection]:

$\forall\ e, t.\quad e \rhd t \rhd t \equiv e \rhd t$

$\forall\ e, t.\quad e \lhd t \lhd t \equiv e \lhd t$

Lemma [A Projection Pair]:

$\forall\ e, t.\ \ e \rhd t \lhd t \preceq e$

Lemma [A Closure Pair]:

$\forall\ e, t.\ \ e \preceq e \lhd t \rhd t$

How to Check?

Part I

Define
$e \in t$

Grand Theorem
$e \in t \iff e \triangleright t$ is crash-free

(related to Blume&McAllester:ICFP'04)

Construct
$e \triangleright t$
(e "ensures" t)

Part II

Simplify ($e \triangleright t$)

Normal form e'

If e' is syntactically safe,
then Done!

Simplification Rules

$$(\lambda x.e_1)\, e_2 \;\Longrightarrow\; e_1[e_2/x] \qquad \text{(BETA)}$$

$$(\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\})\ a \;\Longrightarrow\; \mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to (e_i\ a)\} \quad fv(a) \cap \vec{x_i} = \emptyset \qquad \text{(CASEOUT)}$$

$$\mathsf{case}\ (\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\})\ \mathsf{of}\ alts \;\Longrightarrow\; \mathsf{case}\ e_o\ \mathsf{of}\ \{K_i\ \vec{x_i} \to \mathsf{case}\ e_i\ \mathsf{of}\ alts\}\quad fv(alts) \cap \vec{x_i} = \emptyset \qquad \text{(CASECASE)}$$

$$\mathsf{case}\ K_j \vec{e_j}\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\} \;\Longrightarrow\; \mathtt{UNR} \quad \forall i. K_j \neq K_i \qquad \text{(NOMATCH)}$$

$$\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i; K_j\ \vec{x_j} \to \mathtt{UNR}\} \;\Longrightarrow\; \mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\} \qquad \text{(UNREACHABLE)}$$

$$\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\} \;\Longrightarrow\; e_1 \quad \text{patterns are exhaustive and for all } i, fv(e_i) \cap \vec{x_i} = \emptyset \text{ and } e_1 = e_i \qquad \text{(SAMEBRANCH)}$$

$$\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e\} \;\Longrightarrow\; e_0 \quad e_0 \in \{\mathtt{BAD}\ lbl, \mathtt{UNR}\} \qquad \text{(STOP)}$$

$$\mathsf{case}\ K_i\ \vec{y_i}\ \mathsf{of}\ \{K_i\ \vec{x_i} \to e_i\} \;\Longrightarrow\; e_i[y_i/x_i] \qquad \text{(MATCH)}$$

$$\mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to \ldots \mathsf{case}\ e_0\ \mathsf{of}\{K_i\ \vec{x_i} \to e_i\} \ldots\} \;\Longrightarrow\; \mathsf{case}\ e_0\ \mathsf{of}\ \{K_i\ \vec{x_i} \to \ldots e_i \ldots\} \qquad \text{(SCRUT)}$$

Arithmetic
via External Theorem Prover

```
goo ▷ t_goo = \i ->
              case (i+8 > i) of
              False -> BAD "foo"
              True -> …
```

```
>>ThmProver
i+8>i
>>Valid!
```

```
>>ThmProver
push(i>j)
push(not (j<0))
(i>0)
>>Valid!
```

```
case i > j of
 True -> case j < 0 of
           False -> case i > 0 of
                      False -> BAD "f"
```

Counter-Example Guided Unrolling

```
sumT :: T -> Int
sumT ∈ {x | noT1 x } -> {r | True}
sumT (T2 a) = a
sumT (T3 t1 t2) = sumT t1 + sumT t2
```

After simplifying (sumT ▷ t_{sumT}) , we may have:

```
case (noT1 x) of
True -> case x of
        T1 a -> BAD
        T2 a -> a
        T3 t1 t2 -> case (noT1 t1) of
                    False -> BAD
                    True -> case (noT1 t2) of
                            False -> BAD
                            True -> sumT t1 + sumT t2
```

Step 1:
Program Slicing – Focus on the BAD Paths

```
case (noT1 x) of
True -> case x of
          T1 a -> BAD
          T3 t1 t2 -> case (noT1 t1) of
                        False -> BAD
                        True -> case (noT1 t2) of
                                  False -> BAD
```

Step 2: Unrolling

```
case (case x of
            T1 a -> False
            T2 a -> True
            T3 t1 t2 -> noT1 t1 && noT1 t2) of
True -> case x of
        T1 a -> BAD
        T3 t1 t2 -> case (noT1 t1) of
                    False -> BAD
                    True -> case (noT1 t2) of
                            False -> BAD
```

Counter-Example Guided Unrolling – The Algorithm

$$
\begin{array}{l}
\mathtt{escH}\ rhs\ 0 = \text{“Counter-example :”} +\!\!+\ \mathtt{report}\ rhs \\
\mathtt{escH}\ rhs\ n = \\
\mathtt{let}\ rhs' = \mathit{simplifier}\ rhs \\
\qquad b = \mathtt{noBAD}\ rhs' \\
\mathtt{in\ case}\ b\ \mathtt{of} \\
\quad \mathtt{True} \longrightarrow \text{“No\ \ Bug.”} \\
\quad \mathtt{False} \longrightarrow \mathtt{let}\ s = \mathtt{slice}\ rhs' \\
\qquad\qquad\qquad \mathtt{in\ case\ noFunCall}\ s\ \mathtt{of} \\
\qquad\qquad\qquad\quad \mathtt{True} \longrightarrow \mathtt{let}\ eg = \mathtt{oneEg}\ s \\
\qquad\qquad\qquad\qquad\qquad\quad \mathtt{in}\ \text{“Definite Bug :”} +\!\!+\ \mathtt{report}\ eg \\
\qquad\qquad\qquad\quad \mathtt{False} \longrightarrow \mathtt{let}\ s' = \mathtt{unrollCalls}\ s \\
\qquad\qquad\qquad\qquad\qquad\quad \mathtt{in\ escH}\ s'\ (n-1)
\end{array}
$$

Tracing

(Achieve the same goal as [Meunier, Findler, Felleisen:POPL06]

$g \in t_g$

$g = \ldots$

$f \in t_f$

$f = \ldots g \ldots$

$f \triangleright t_f = \ldots g \triangleleft t_g \ldots$

Inside "g" lc $(g \triangleleft t_g)$

```
case fin p[e/x] of
  True  -> e
  False -> BAD "f"
```

$(\backslash g \to \ldots g \ldots) \triangleright t_g \to t_f$

Counter-Example Generation

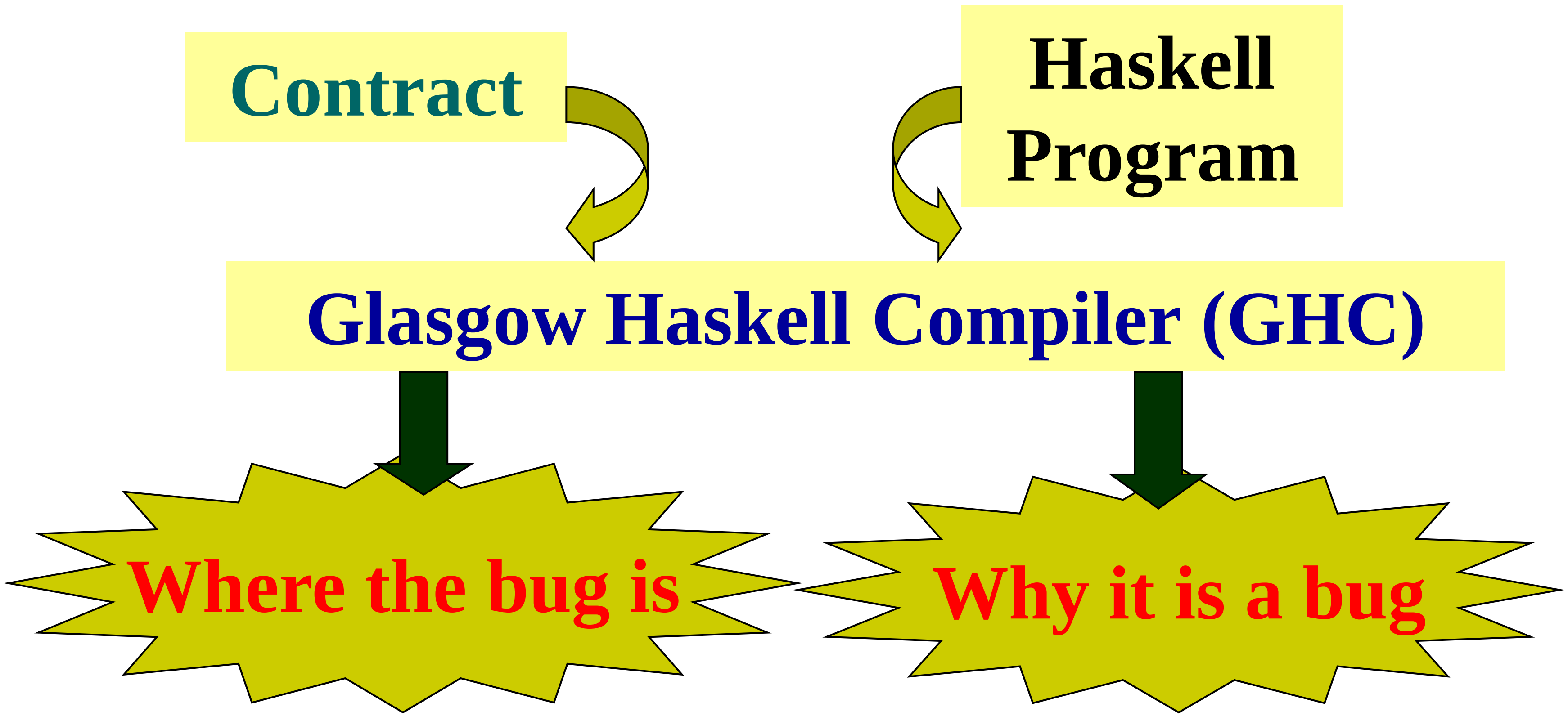
```
f1 ∈ x:Ok -> { x < z } -> Ok
f2 x z = 1 + f1 x z
```

```
f3 [] z = 0
f3 (x:xs) z = case x > z of
                True -> f2 x z
                False -> ...
```

```
f3 ▷ Ok = \xs -> \z ->
 case xs of
 [] -> 0
 (x:y) -> case x > z of
          True -> Inside "f2" <l2>
                       (Inside "f1" <l1> (BAD "f1"))
          False -> …
```

```
Warning <l3>: f3 (x:y) z where x>z
              calls f2
              which calls f1
              which may fail f1's precondition!
```

Conclusion



Contract
Haskell Program
Glasgow Haskell Compiler (GHC)
Where the bug is
Why it is a bug

Summary

- Static contract checking is a fertile and under-researched area
- Distinctive features of our approach
  - Full Haskell in contracts; absolutely crucial
  - Declarative specification of "satisfies"
  - Nice theory (with some very tricky corners)
  - Static proofs
  - Modular Checking
  - Compiler as theorem prover

Contract Synonym

```
contract Ok = {x | True}
contract NonNull = {x | not (null x)}

head :: [Int] -> Int
head ∈ NonNull -> Ok
head (x:xs) = x

{-# contract Ok = {x | True} -#}
{-# contract NonNull = {x | not (null x)} #-}
{-# contract head :: NonNull -> Ok #-}
```

Actual Syntax

Recursion

```
f ▷t
=\f->f ▷ t->t
= …
=(… (f ◁ t)…) ▷ t
```

Suppose `t = t1 -> t_2`

```
f ▷ t1 -> t2
= \f->f ▷(t1 -> t2) -> (t1 -> t2)
= …
=(… (f ◁ t1 -> t2)…) ▷ t1 -> t2
=\v2.((…(\v1.((f (v1 ▷t1)) ◁ t2)) (v2◁ t1) …) ▷ t2)
```