Gustave, ses pompes et ses œuvres

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Story of a function

- What is a function ?
- it may mean set-theoretical graph, like in crazy Bourbaki
- it may mean computable function
- it may mean partial computable function
- it may mean algorithm

Algorithm

- algorithm mean computable by a sequential process
- it means programmable in CDS



- CDS = Concrete Data Structures (Berry-Curien, 1982).
- It builds on Kahn (IFIP 1971)
- And on Vuillemin 1973, Lévy 1976, Kahn-MacQueen 1976
- And then on Kahn-Plotkin 1978, Berry 1979, Berry-Lévy

Those were the days







Where is the G key?



More on CDS

- The slogan of CDS was "the programming language of the 90's"
- It was more than 30 years ahead of its time
- What Nagārjuna has to say about time
- "Ainsi le futur vient en premier et le passé après"
- What Gégé has to say about time
- "Tout se passe au même moment, mais dans le bon ordre"

Headache

A function's story

- Q: What function is it ?
- A: Gustave's function
- Q: Is it an interesting function ?
- A: Not really
- Q: Then, is it an algorithm ?
- A: Neither

Definition of Gustave's fonction

- $GG(x, 1, 0) = \top$
- $GG(\mathbf{0}, x, \mathbf{1}) = \top$
- $GG(1,0,x) = \top$

Discussion

- The associated function is semi-trivial
- Its value is always \top
- But it is not total (even on $\{0,1\}$)
- And thus GG(A,B,C) is not easy to compute in a top-down manner
- The problem is: which of A,B,C should I compute on first ?
- Is parallel evaluation unavoidable?

Plotkin already showed this with parallel or?



- OR(x,1) = 1
- OR(1,x) = 1
- OR(0,0) = 0 needs parallel evaluation as well

Counter-objection

- Parallel OR is unstable
- It is not orthogonal, it admits critical pairs
- Gustave's function is stable
- but it is not Vuillemin sequential
- Thus it is a much more subtle computational paradox

Oh, really ?



Why is it called Gustave's function ?

Because I called it so ! A pure gratuitous act A Gustave function act A proof of existence

Historical details

Gustave surfaced in my 1985 course notes "Deduction and Computation". These notes were published first in "Fundamentals in Artificial Intelligence", Eds. W. Bibel and Ph. Jorrand, Springer LNCS 232 (1986) pp. 39-74. Also presented at the International Summer School on Logic of Programming and Calculi of Discrete Design, Marktoberdorf, Aug. 86. Published in "Logic of Programming and Calculi of Discrete Design", Ed. M. Broy, NATO ASI Series, F-36, Springer-Verlag 1987, 305-342. Also in INRIA Research Report RR-513, 1986, easily found on

the HAL free archive by Googling 'Gustave function'.

Original citation (1985)

There I say, as a commentary to the **Standardization theorem**:

"This theorem suggests it can be used to define a *computation rule*, usable to drive an interpreter computing 'lazily'. However, this is not true, since the standard derivation in a derivation class is not simply a function of its starting term. For instance, consider **Gustave's function**, an example due to G.G. Berry: ..."

Killing two birds with one stone, by revealing in one sentence two nicknames of Gérard.

This also appeared in my CMU Course notes "Formal Structures for Computation and Deduction", May 1986, p. 94. The bait was set, I only had to wait. This took 5 years.

Some of my students at my CMU class in 1985

- Boris Trakhtenbrot
- Dana Scott
- Rick Statman
- John Reynolds
- Thierry Coquand
- Daniel Leivant
- Steeve Brookes
- Christine Paulin
- Philippe Le Chenadec

and Jean-Jacques Lévy was guest lecturer.



Gustave reappears in publication in 1991

This is unique so far. A proper quotation:

"This is a variant of 'Gustave's function' (attributed to Berry

[11] by Huet [9]);". In "Towards a Theory of Parallel

Algorithms on Concrete Data Structures", by S. Brookes and S. Geva, TCS, June 1991.

where indeed, in the bibliography, is listed:

[9] Huet, G., Formal Structures for Computation and

Deduction, Class notes for graduate course at CMU, May, 1986.

A perfect hoax

From then on, my tracks are lost. Nobody knows anymore the origin of Gustave's function, not even Steve Brookes, in spite of his 'smoking gun' paper.

A long history of plagiarism

This list of anonymous quotations reads like a Who's who of theoretical computer science:

- "Bool be (a variation on) Gustave's function (due to Berry)" in "Stable and Sequential Functions on Scott domains", by S. Brookes and S. Geva, CMU course notes CS-92-121, June 1992.
- "The Gustave function is an example of a function which is stable but not Vuillemin Sequential". In Course notes EE290N, Berkeley U., Edward A. Lee, Oct 1996.
- "The Gustave function. This fantastic counterexample is due to G. Berry ~ 1978." ... "The case of Gustave is much more delicate: Ehrard found a beautiful generalization of coherent spaces, *hypercoherences*, which manages to kill Gustave's function, ..." in Jean-Yves Girard "On the meaning of logical rules I, Jan. 98".

By 1999 it had become classic in semantics publications

- "Classically, this condition excludes the parallel-or function, Berry's Gustave function, etc. [Vui73, KP78, Cur86]". In "Concurrent interaction games" by Samson Abramsky, 1999.
- "A more subtle example was given by Berry and is known as Gustave's function, or Berry-Kleene function (as Kleene had encountered a variant of this function too):". In P. L. Curien, "Symmetry and interactivity in programming", Oct. 2001.
- Wrong terminology: "BK(x,y,z) (footnote: Another name that is used for this function is *gustave*)". In Roberto M. Amadio and Pierre-Louis Curien, "Domains and Lambda-Calculi", Cambridge U. Press, 1998.
- "Berry's 'Gustave' functions,". In "Softness of Hypercoherences and MALL Full Completeness" by Richard Blute, Masahiro Hamano and Philip Scott, Nov. 2003.

By 2007 Even GG is forgotten!

- "In fact there is a PS, the Gustave one, which is the proof theoretical counterpart of the Gustave function G". In "A Characterization of Hypercoherent Semantic Correctness in Multiplicative Additive Linear Logic" by P. Tranquilli, 2007.
- "Undefinability of Gustave's function". In "Reduction under Substitution", Joörg Endrullis and Roel de Vrijer, July 2008.
- "Gustave's function:" in "Sur la λ -définissabilité", A. Bucciarelli, Dec. 2009.
- Now Gustave's function is autonomous. Gustave made History. People are proud of attributing new notions to Gustave.

Freudian analysis of the Gustave phenomenon

The World (of researchers in TCS) according to Girard.

- According to Girard, our professional world is divided into three categories of people:
- The *fantastic* ones, who are proud of attributing new paradoxes to Gustave, obeying their instinct of pleasure.
- The *beautiful* ones, who try to kill Gustave with hypercohences or whatever, obeying their instinct of death.
- The ugly ones, who gesticulate with Broccoli logic.
- Similar to Sergio Leone's classification: The Good, the Bad and the Ugly.

I hate the style of this talk



Who invented Gustave's function?

We abstract Gustave's function to its essence.

Definition (Huet 2011). A *Gustave function* over some domain is a stable function that is not sequential.

The Gustave function inventor



The Gustave function inventor

Marie Jean Antoine Nicolas de Caritat

Marquis de Condorcet

(17 September 1743 - 28 March 1794)

Nicolas de Condorcet

- Glorious actor of the French Revolution
- Was the main intellectual leader as Geometer Academician
- Was a democrat in favor of universal suffrage
- Including jews, who should be counted as citizens
- Including niggers, who should be freed
- Including women, who should have equal rights to men
- Asked for free laic education irrespectively of class
- Was against the death penalty as a matter of principle
- Hence wanted the King to be 'only' served a life sentence at the galleys
- Hence was denounced as a traitor, jailed and murdered in his cell

Condorcet's temper

- Condorcet was atrabilious and bad-tempered
- He could not stand jerks
- He would put them in their place, publically
- Thus had lots of ennemies
- He calmed himself down by doing maths

Condorcet as a mathematician

- He was a professional mathematician
- He was a friend of d'Alembert and Lagrange
- He corresponded with Euler
- Believed that social reforms ought to be scientifically grounded
- Thus studied voting processes
- Was a precursor of Game Theory
- Proved the Jury Theorem
- Invented the "Condorcet paradox"

The Condorcet paradox (1785)

Assume three candidates, A, B and C postulate for a position. Voters submit linear orderings of $\{A,B,C\}$ as votes. A social choice function selects a winner as a function of the votes. When preferences are inconsistent, every choice function will make a majority of voters unhappy as contradicting their vote.

The Condorcet paradox with 3 voters

- Voter 1 prefers A to B to C.
- Voter 2 prefers B to C to A.
- Voter 3 prefers C to A to B.
- No matter who is selected, 2/3 of the voters will be unhappy!
- It is not always possible for a social choice function to pick a candidate that will beat all other candidates in pairwise comparisons.
- A Condorcet winner is one that wins all such votes.
- The Condorcet paradox exhibits a Gustave function.

Ballot theory

Let $C = \{c_1, ..., c_n\}$ be the set of candidates, $V = \{v_1, ..., v_{2N+1}\}$ the set of voters.

We associate to each voter $v \in V$ his ballot as a total ordering $>_v$ on C. Thus $a >_v b$ iff v ranks a before b. We extend these orderings to V by majority:

$$a >_V b \equiv |\{v \in V | a >_v b\}| > N.$$

We say that c beats candidate c' if $c >_V c'$, and that c defeats c' iff $c >_V c'$ and for any $c'' c' >_V c'' \Rightarrow c >_V c''$. A Condorcet winner is $>_V$ -maximal:

$$W(w) \Leftrightarrow \forall c \in C \ w = c \lor w >_V c.$$

We define GT(c, c') = 1 if $c >_V c'$, 0 if $c' >_V c$, \perp if c = c'. Let $S = \{s_1, ..., s_p\}$ be obtained from C by recursively removing all defeated candidates.

We define the function $G \in B^p_{\perp} \to U$, where $B = \{0, 1\}$ and $U = \{\perp, \top\}$, by: $G(b_1, ..., b_p) = \top \Leftrightarrow \exists s \in S \ b_i = GT(s, s_i).$

Gustave Condorcet

The function G is extended by monotonicity. It is stable because candidates in S are mutually undefeated, and their defining clauses are thus incomparable. If there exists a Condorcet winner, it defeats all others, p = 1, and G is the constant function defined by $G(\perp) = \top$. Otherwise, G is not sequential at \perp . Indeed, no s in S may be an index, since its defining clause refutes the fact. Thus any vote which does not admit a Condorcet winner defines a Gustave function. Indeed, the Condorcet paradox gives $S = \{A, B, C\}$ with $A >_V B$, $B >_V C$ and $C >_V A$, p = 3, and here G = GG. Furthermore, if A and B do not defeat each other, with say $A >_V B$, there must exist C unique such that $B >_V C$ and $C >_V A$ and thus GG is the only Gustave function definable by voting paradoxes.

This should be expected, GG is clearly unique!

Perrin's statue of Gustave Condorcet



Implementing democracy according to Condorcet

Condorcet proposed to implement democracy as a 3 step process. First the electors vote. Then the social process selects a winner. Then we verify that the social process is not tyrannic by running the winner through a consensus checker. The Condorcet paradox exibits a bug in the democratic process based on a notion of consensus demanding of a winner to be Condorcet compliant. When there is no Condorcet winner, any candidate is at best non defeated, but would be beaten by some other undefeatable one in a binary election.

Thus new democratic processes must be coined, in order to avoid electoral deadlocks. For instance, dictatorship is fine, and avoids the cost of running elections.

Gustave won the Nobel Prize

For elections with 3 or more candidates, there is no social choice function that satisfies 3 properties:

- Non Dictatorship
- Unanimity (Pareto Efficiency)
- Independence of Irrelevant Alternatives

Arrow received the Nobel Price in 1972 for this Gustave Function!

Gustavology is a promising new field.

But beware of Gustave Broccoli !

The Goa theorem G. Berry, G. Huet & J. Vuillemin (Goa, Dec. 1989)

Life is a Gustave function.

Yes, we can !



Thanks

- to Philippe Flajolet who coined Gustave for α -conversion of Gérard
- to Gérard Berry and Jean-Jacques Lévy for their fundamental contributions to the theory of computation and many happy memories of interaction
- to Thierry Coquand who noticed the Gustave character of Condorcet