Towards a separation logic for Multicore OCaml

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The weak memory model

Multicore OCaml

Extension of the OCaml language with **multicore programming**.

Research project at OCaml Labs (Cambridge), will be merged eventually.

Strengths:

- brings multicore abilities to a functional, statically typed, memory-safe programming language;
- (gives the programmer a simpler memory model than that of C11, hopefully;)
- limited performance drop for sequential code.

Goals of this PhD:

- Build a proof system for Multicore OCaml programs.
- Prove interesting concurrent data structures.

Weak memory models

Sequential consistency is unrealistic.

We need a **weaker memory model**, where different threads have different **views** of the shared state.

The model should be specific to our language.

Existing works: Java (2000s), C11 (2010s; also Rust).

Candidate model for Multicore OCaml:

Dolan, Sivaramakrishnan, Madhavapeddy.
Bounding Data Races in Space and Time.
PLDI 2018.

Two access modes: non-atomic, atomic.

An operational model for Multicore OCaml: non-atomics

$$
\begin{array}{l} \texttt{x := } x_1 \\ \texttt{A := !y} \end{array}
\;\Bigg\|\;
\begin{array}{l} \texttt{y := } y_1 \\ \texttt{B := !x} \end{array}
$$

An operational model for Multicore OCaml: non-atomics

$$\begin{array}{l|l} \texttt{x := } x_1 & \texttt{y := } y_1 \\ \texttt{A := !y} & \texttt{B := !x} \end{array}$$

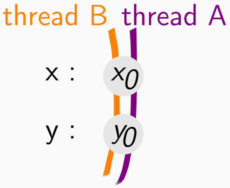
Each **non-atomic** location has a **history**, *i.e.* a map from timestamps to values (timestamps are per location).

x : x_0

y : y_0

An operational model for Multicore OCaml: non-atomics

$$\begin{array}{l} \texttt{x := } x_1 \\ \texttt{A := !y} \end{array} \;\Big\Vert\; \begin{array}{l} \texttt{y := } y_1 \\ \texttt{B := !x} \end{array}$$

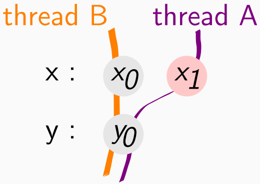


Each **non-atomic** location has a **history**, *i.e.* a map from timestamps to values (timestamps are per location).

Each thread has its own **view** of the non-atomic store, *i.e.* a map from non-atomic locations to timestamps.

An operational model for Multicore OCaml: non-atomics

`x := ` x_1	`y := ` y_1
`A := !y`	`B := !x`



non-atomic write

- Timestamp must be fresh.
- Timestamp must be newer than current thread's view.
- Current thread's view is updated.

An operational model for Multicore OCaml: non-atomics

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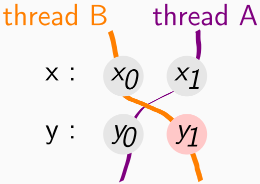

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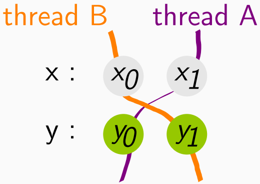
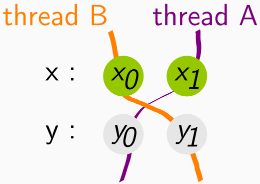


non-atomic read

- Returns **any** value at least as recent as current thread's view.
- Current thread's view is unchanged.

An operational model for Multicore OCaml: non-atomics

```
x := x1  ||  y := y1
A := !y  ||  B := !x
```



non-atomic read

- Returns **any** value at least as recent as current thread's view.
- Current thread's view is unchanged.

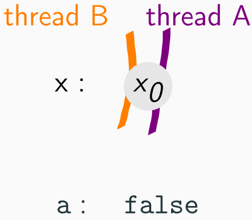
An operational model for Multicore OCaml: atomics

Non-atomic locations are useful for updating the state locally, but they don't provide synchronization.

Atomic locations allow the message-passing idiom.

An operational model for Multicore OCaml: atomics

$\texttt{x} := x_1$	$\texttt{REPEAT}$
$\texttt{a} :=_{\texttt{at}} \texttt{true}$	$\quad \texttt{C} := \texttt{!}_{\texttt{at}}\ \texttt{a}$
	$\texttt{UNTIL C = true}$
	$\texttt{B := !x}$



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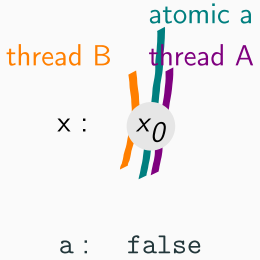
	UNTIL C = true
	B := !x



Each **atomic** location stores **one** value, and **one view** of the non-atomic store.

An operational model for Multicore OCaml: atomics

x := x_1	REPEAT
a $:=_{\mathrm{at}}$ true	C := $!_{\mathrm{at}}$ a
	UNTIL C = true
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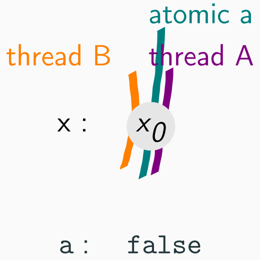

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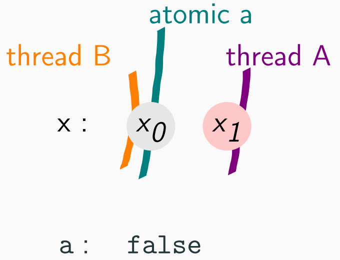
`x := ` x_1
`a :=`$_{at}$ `true`
	`REPEAT`
	`  C := !`$_{at}$ `a`
	`UNTIL C = true`
	`B := !x`



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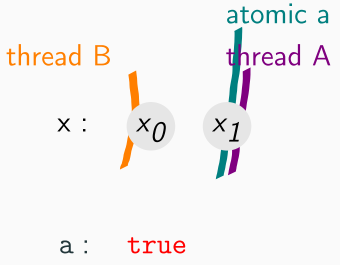
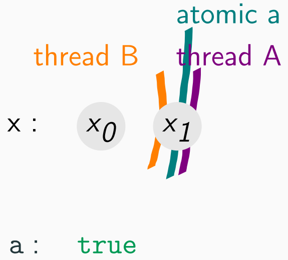


atomic write

Merges the writer's view into the atomic location's view.

An operational model for Multicore OCaml: atomics

```
x := x1          ||  REPEAT
a :=at true      ||    C := !at a
                 ||  UNTIL C = true
                 ||  B := !x
```

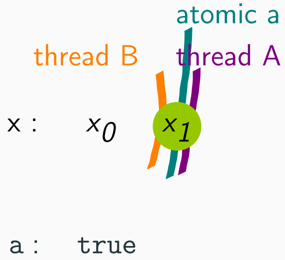


atomic read

Merges the atomic location's view into the reader's view.

An operational model for Multicore OCaml: atomics

```
x := x₁            ||  REPEAT
a :=_at true       ||     C := !_at a
                   ||  UNTIL C = true
                   ||  B := !x
```



Our program logic

Rules of non-atomic locations

The predicate $x \mapsto v$ means that we own the non-atomic location x and that we have **seen** its latest value, which is v.

Non-atomic write:

$$\{x \mapsto v\}$$
$$x := v'$$
$$\{\lambda().\ x \mapsto v'\}$$

Non-atomic read:

$$\{x \mapsto v\}$$
$$!\,x$$
$$\{\lambda v'.\ v' = v\ *\ x \mapsto v\}$$

Impact of the weak memory model on our CSL

Invariants are the mechanism by which threads can share propositions in a Concurrent Separation Logic such as Iris:

$$\frac{\{P \ast I\}\, e\, \{Q \ast I\} \quad e \text{ atomic}}{\boxed{I} \vdash \{P\}\, e\, \{Q\}}$$

The proposition $x \mapsto v$ is **subjective**: its truth depends on the thread's view of memory.

It is unsound to share it *via* an invariant.

Propositions which are true in all threads are called **objective**:

- "pure" facts, such as $v = 5$
- ghost state, such as $\gamma \hookrightarrow \circ 5$
- atomic state, such as $a \mapsto_{\mathsf{at}} (v, \mathcal{V})$

Only objective propositions can be put in an invariant.

Rules of atomic locations (simplified)

The predicate $a \mapsto_{\text{at}} v$ means that we own the atomic location a, which stores the value v.

It is **objective**.

Atomic write:

$$\{a \mapsto_{\text{at}} v\}$$
$$a :=_{\text{at}} v'$$
$$\{\lambda().\ a \mapsto_{\text{at}} v'\}$$

Atomic read:

$$\{a \mapsto_{\text{at}} v\}$$
$$!_{\text{at}}\ a$$
$$\{\lambda v'.\ v' = v \ast a \mapsto_{\text{at}} v\}$$

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Atomic read:

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$$!_{\mathsf{at}}\ a$$
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views

Views are ordered by inclusion.

The predicate $\uparrow \mathcal{V}$ means "the current thread's view includes $\mathcal{V}$".

Rules of atomic locations

The predicate $a \mapsto_{\text{at}} (v, \mathcal{V})$ means that we own the atomic location a, which stores the value v and a view (at least) $\mathcal{V}$.

It is **objective**.

Atomic write:

$$\{a \mapsto_{\text{at}} (v, \mathcal{V}) \ * \ \uparrow \mathcal{V}'\}$$
$$a :=_{\text{at}} v'$$
$$\{\lambda().\ a \mapsto_{\text{at}} (v', \mathcal{V}')\}$$

Atomic read:

$$\{a \mapsto_{\text{at}} (v, \mathcal{V})\}$$
$$!_{\text{at}}\ a$$
$$\{\lambda v'.\ v' = v \ * \ a \mapsto_{\text{at}} (v, \mathcal{V}) \ * \ \uparrow \mathcal{V}\}$$

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It is **objective**.

Atomic write:

$$\{a \mapsto_{\mathsf{at}} (v, \mathcal{V}) \ast \uparrow \mathcal{V}'\}$$
$$a :=_{\mathsf{at}} v'$$
$$\{\lambda().\ a \mapsto_{\mathsf{at}} (v', \mathcal{V}' \sqcup \mathcal{V}) \ast \uparrow \mathcal{V}\}$$

views

Views are ordered by inclusion.

The predicate $\uparrow \mathcal{V}$ means "the current thread's view includes $\mathcal{V}$".

Atomic read:

$$\{a \mapsto_{\mathsf{at}} (v, \mathcal{V})\}$$
$$!_{\mathsf{at}}\ a$$
$$\{\lambda v'.\ v' = v \ast a \mapsto_{\mathsf{at}} (v, \mathcal{V}) \ast \uparrow \mathcal{V}\}$$

The message passing idiom

The objective proposition "P at $\mathcal{V}$" is the subjective proposition P seen at a fixed view $\mathcal{V}$.

$$P \Longleftrightarrow \exists \mathcal{V}.\ (\uparrow \mathcal{V}) \ast (P \text{ at } \mathcal{V})$$

($\Rightarrow$) If P holds now, then it holds at the current view.

($\Leftarrow$) If P holds at some earlier view, then it holds now.

The message passing idiom

$$P \iff \exists \mathcal{V}.\ (\uparrow \mathcal{V}) \ * \ (P \,\mathsf{at}\, \mathcal{V})$$

$\{ \qquad\qquad P\}$

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The message passing idiom

$$P \iff \exists \mathcal{V}.\ (\uparrow \mathcal{V}) * (P \,\mathsf{at}\, \mathcal{V})$$

$\{a \mapsto_{\mathsf{at}} (\mathtt{false}, \varnothing) * P\}$	
	$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P\}$

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$\{a \mapsto_{\mathsf{at}} (\mathtt{false}, \varnothing) \ast P \text{ at } \mathcal{V} \ast \uparrow \mathcal{V}\}$

$\mathtt{a}\ :=_{\mathsf{at}}\ \mathtt{true}$

$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) \ast P \text{ at } \mathcal{V}\}$

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The message passing idiom

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`a :=`$_{\mathsf{at}}$ `true`

$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) \ast P \text{ at } \mathcal{V}\}$

This proposition is objective:
it can be put in an invariant.

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$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P \text{ at } \mathcal{V}\}$ $\xrightarrow{\text{inv}}$ $\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P \text{ at } \mathcal{V}\}$

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The message passing idiom

$$P \iff \exists \mathcal{V}.\ (\uparrow \mathcal{V}) * (P \,\mathsf{at}\, \mathcal{V})$$

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`a :=`$_{\mathtt{at}}$ `true`

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inv

$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P \,\mathsf{at}\, \mathcal{V}\}$

`C := !`$_{\mathtt{at}}$ `a`

$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P \,\mathsf{at}\, \mathcal{V} * \uparrow \mathcal{V}\}$

$\{a \mapsto_{\mathsf{at}} (\mathtt{true}, \mathcal{V}) * P\}$

This proposition is objective:
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Example: spin lock

A spin lock implements a lock using an atomic boolean variable.

```
let rec acquire lk =
  if CAS lk false true
  then ()
  else acquire lk
```

```
let release lk =
  lk :={at} false
```

The invariant in the sequentially consistent model is:

$$\mathsf{lockInv}\ \mathtt{lk}\ P \triangleq$$

$$\mathtt{lk} \mapsto_{\mathsf{at}} \mathtt{true} \ \vee\ (\quad \mathtt{lk} \mapsto_{\mathsf{at}} \mathtt{false} \quad * \ P)$$

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  else acquire lk
```

```
let release lk =
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```

The invariant in the weak model is:

$$\text{lockInv}\ \texttt{lk}\ P \triangleq$$

$$\texttt{lk} \mapsto_{\mathsf{at}} \texttt{true} \quad \vee \quad (\exists \mathcal{V}.\ \texttt{lk} \mapsto_{\mathsf{at}} (\texttt{false}, \mathcal{V}) * P \ \mathsf{at}\ \mathcal{V})$$

Example: ticket lock

A ticket lock implements a lock using two atomic integer variables.

The invariant in the sequentially consistent model is:

$$
\begin{aligned}
&\mathsf{lockInv}\ \texttt{turn}\ \texttt{next}\ \gamma\ P \triangleq \\
&\quad \exists t, n. \\
&\qquad\quad \texttt{turn} \mapsto_{\mathsf{at}} t \\
&\qquad * \ \texttt{next} \mapsto_{\mathsf{at}} n \\
&\qquad * \ (\mathsf{ticket}\ \gamma\ t \vee (\mathsf{locked}\ \gamma\ *\ P)) \\
&\qquad * \ \gamma \hookrightarrow (\bullet\ \ldots)
\end{aligned}
$$

Example: ticket lock

A ticket lock implements a lock using two atomic integer variables.

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\begin{aligned}
&\mathsf{lockInv}\ \mathtt{turn}\ \mathtt{next}\ \gamma\ P \triangleq \\
&\quad \exists t, n, \mathcal{V}. \\
&\qquad\quad \mathtt{turn} \mapsto_{\mathsf{at}} (t, \mathcal{V}) \\
&\qquad * \ \mathtt{next} \mapsto_{\mathsf{at}} n \\
&\qquad * \ (\mathsf{ticket}\ \gamma\ t \vee (\mathsf{locked}\ \gamma\ *\ P\ \mathsf{at}\ \mathcal{V})) \\
&\qquad * \ \gamma \hookrightarrow (\bullet \ldots)
\end{aligned}
$$

Example: Dekker's mutual exclusion

Dekker's algorithm solves the mutual exclusion problem using three atomic variables.

The invariant and representation predicate in the SC model are:

$$\begin{aligned}
&\mathsf{DekkerInv}\ \mathtt{turn}\ \mathtt{flag}_0\ \mathtt{flag}_1\ \gamma\ P \triangleq \\
&\quad \exists t, f_0, f_1, c_0, c_1. \\
&\qquad \left(\forall i \in \{0,1\}.\ \mathtt{flag}_i \mapsto_{\mathsf{at}} f_i \qquad * \ \gamma_i \hookrightarrow \bullet\, c_i\right) \\
&\qquad * \ ((\neg c_0 \wedge \neg c_1) \mathrel{-\!\!*} P) \\
&\qquad * \ \ldots \\
&\mathsf{isDekker}\ i\ \gamma \triangleq \\
&\qquad \gamma_i \hookrightarrow \circ\, \mathtt{false}
\end{aligned}$$

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&\quad \exists t, f_0, f_1, c_0, c_1, \mathcal{V}_0, \mathcal{V}_1. \\
&\qquad\quad \big(\forall i \in \{0,1\}.\ \mathtt{flag}_i \mapsto_{\mathsf{at}} (f_i, \mathcal{V}_i) \ast \gamma_i \hookrightarrow \bullet\, c_i \ast \gamma'_i \hookrightarrow \bullet\, \mathcal{V}_i\big) \\
&\qquad \ast\ ((\neg c_0 \wedge \neg c_1) \twoheadrightarrow\!\!\ast\ P\ \mathsf{at}\ (\mathcal{V}_0 \sqcup \mathcal{V}_1)) \\
&\qquad \ast\ \ldots \\
&\mathsf{isDekker}\ i\ \gamma \triangleq \\
&\quad \exists \mathcal{V}. \gamma_i \hookrightarrow \circ\,\mathtt{false} \ast \gamma'_i \hookrightarrow \circ\,\mathcal{V} \ast \uparrow \mathcal{V}
\end{aligned}$$

Model of the logic in Iris

Propositions are predicates on views:

$$\mathrm{vProp} \triangleq \mathrm{view} \longrightarrow \mathrm{iProp}$$

$$\uparrow \mathcal{V}_0 \triangleq \lambda \mathcal{V}.\ \mathcal{V}_0 \sqsubseteq \mathcal{V}$$

$$P \mathrel{-\!\!*} Q \triangleq \lambda \mathcal{V}\ . \qquad\qquad P\ \mathcal{V} \mathrel{-\!\!*} Q\ \mathcal{V}$$

Model of the logic in Iris

Propositions are **monotonic** predicates on views:

$$\mathrm{vProp} \triangleq \mathrm{view} \xrightarrow{\mathsf{mon}} \mathrm{iProp}$$

$$\uparrow \mathcal{V}_0 \triangleq \lambda \mathcal{V}.\ \mathcal{V}_0 \sqsubseteq \mathcal{V}$$

$$P \mathrel{-\!\!*} Q \triangleq \lambda \mathcal{V}_1.\ \forall \mathcal{V} \sqsupseteq \mathcal{V}_1.\ P\ \mathcal{V} \mathrel{-\!\!*} Q\ \mathcal{V}$$

Model of the logic in Iris

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We equip a language-with-view with an operational semantics:

$$\mathsf{exprWithView} \triangleq \mathsf{expr} \times \mathsf{view}$$

Iris builds a WP calculus for exprWithView in iProp.

We derive a WP calculus for expr in vProp and prove adequacy:

$$\mathsf{WP}\ e\ \varphi \triangleq \lambda \mathcal{V}_1.\ \forall \mathcal{V} \sqsupseteq \mathcal{V}_1.\ \mathsf{WP}\ \langle e, \mathcal{V} \rangle\ \left(\lambda \langle v, \mathcal{V}' \rangle.\ \varphi\ v\ \mathcal{V}'\right)$$

where $\varphi : \mathsf{val} \rightarrow \mathsf{vProp}$

Future work

Plans for the future:

- Prove more elaborate shared data structures
  - *e.g.* bounded queues with a circular buffer
- Data races on non-atomics:
  - How to allow them?
  - What are they useful for?