

# SMTCoq: safe and efficient automation in Coq

Chantal Keller

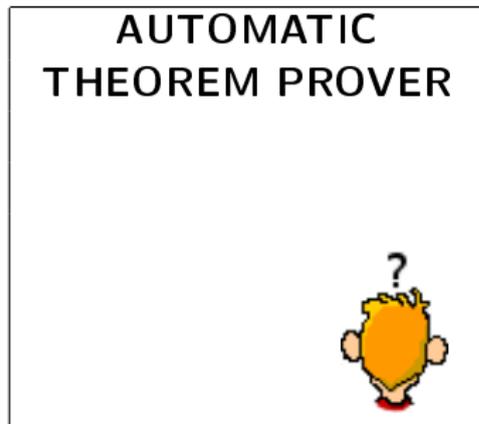
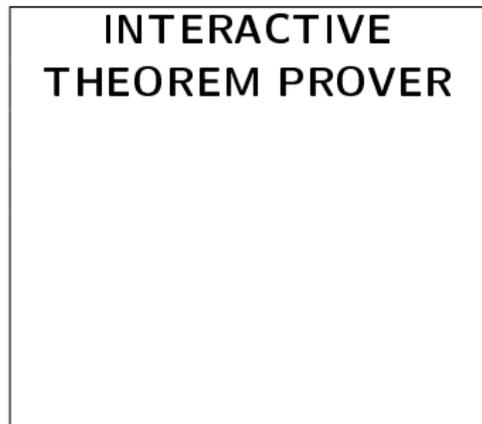
LRI, Univ. Paris-Sud, Université Paris-Saclay

Joint work with École polytechnique, Inria Sophia-Antipolis,  
Iowa University, Université Paris-Diderot

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# Motivation



# SMT and Coq in a nutshell

	SAT/SMT solvers	Coq
Expressivity	First-order logic	CIC
Safety	Trust the whole software	Kernel for proof checking
Automation	Automatic proof	User-guided proof

↔ benefit from both worlds

## Application 1: combinatorial mathematics

### Erdős Discrepancy Conjecture

For any infinite sequence  $\langle x_1, x_2, \dots \rangle$  of  $\pm 1$  integers and any integer  $C$ , there exist integers  $k$  and  $d$  such that

$$\left| \sum_{i=1}^k x_{i \times d} \right| > C$$

Proof for some  $C_0$ :

Find  $l$  such that

$$\forall \langle x_1, x_2, \dots, x_l \rangle, \forall k, \forall d, \left| \sum_{i=1}^k x_{i \times d} \right| \leq C_0$$

is unsatisfiable.

## In Coq: computational reflection with efficient data-structures

```
Lemma erdos60th :  
  forall rho, ~ valid (interp_var rho) erdos60.  
Proof.  
  apply (@checker_correct erdos60 ("erdos1161_60.reso")).  
  native_cast_no_check (refl_equal true).  
Qed.
```

```
Lemma Erdos (x_ : nat → bool) :  
  exists k, exists d,  
    '|(\sum_(1 <= i < k) [x_ (i * d)])| > 2.  
Proof.  
  ...  
  apply erdos60th.  
  ...  
Qed.
```

(Credits: École polytechnique, Inria Sophia-Antipolis)

## Application 2: tedious Coq proofs

```
Variable e : G, inv : G → G, op : G → G → G.
```

```
Hypothesis associative :
```

```
  forall a b c, op a (op b c) = op (op a b) c.
```

```
Hypothesis identity : forall a, (op e a = a).
```

```
Hypothesis inverse : forall a, (op (inv a) a = e).
```

```
Add_lemmas associative identity inverse.
```

```
Lemma identity' :
```

```
  forall a, (op a e = a).
```

```
Proof. smt. Qed.
```

```
Lemma inverse' :
```

```
  forall a, (op a (inv a) = e).
```

```
Proof. smt. Qed.
```

```
Lemma unique_identity e' :
```

```
  (forall z, op e' z = z) → e' = e.
```

```
Proof. intros pe'; smt pe'. Qed.
```

```
Clear_lemmas.
```

(Credits: Iowa University,  
Université Paris-Diderot,  
Univ. Paris-Sud)

## Example of CompCert

“Most of the development is conducted in first-order logic, suggesting the possibility of using automated theorem provers such as SMT solvers. Preliminary experiments with using SMT solvers to prove properties of the CompCert memory model indicate that many but not all of the lemmas can be proved automatically. While fully automated verification of a program like CompCert appears infeasible with today’s technology, we expect that our interactive proof scripts would shrink significantly if Coq provided a modern SMT solver as one of its tactics.”



# A fragment of CompCert revisited with SMTCoq

```

Variable block , mem : Set .
Variable alloc_block : mem → Z → Z → block .
Variable alloc_mem : mem → Z → Z → mem .
Variable valid_block : mem → block → bool .

Hypothesis alloc_valid_block_1 m lo hi b :
  valid_block (alloc_mem m lo hi) b →
    ((b =? (alloc_block m lo hi)) || valid_block m b) .

Hypothesis alloc_valid_block_2 m lo hi b :
  ((b =? (alloc_block m lo hi)) || valid_block m b) →
    valid_block (alloc_mem m lo hi) b .

Hypothesis alloc_not_valid_block m lo hi :
  negb (valid_block m (alloc_block m lo hi)) .

Lemma alloc_valid_block_inv m lo hi b :
  valid_block m b → valid_block (alloc_mem m lo hi) b .
Proof. intro H. smt alloc_valid_block_2 H. Qed.

Lemma alloc_not_valid_block_2 m lo hi b' :
  valid_block m b' → b' =? (alloc_block m lo hi) = false .
Proof. intro H. smt alloc_not_valid_block H. Qed.

```

# Challenges

- combinatorial mathematics: efficiency
- tedious Coq proofs: expressivity, encodings
- in addition: modularity with respect to solvers and theories

## SMTCoq: in one system

- provers: ZChaff, glucose, veriT, CVC4
- “theories”: equality, linear arithmetic, bit vectors, arrays, quantified hypotheses
- able to handle large combinatorial proofs
- Coq tactics that can combine theories

## SMTCoq: skeptical approach

### Certified ATP:

- prove the correctness of the code of the ATP
- + once and for all
- + completeness possible
  - not flexible nor modular; freezes an implementation
  - hard

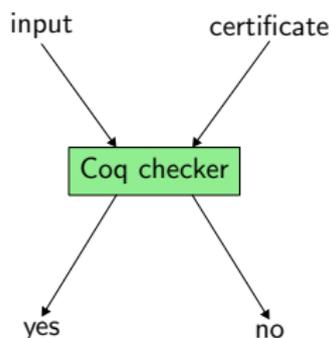
### Certifying ATP:

- the ATP gives certificates that can be checked
  - certificates to check each time (but efficient)
  - no completeness (or at the meta-level)
- + very flexible and modular
- + easier (certified checker)

# Outline

- 1 Skeptical interaction with external provers
- 2 Checker
- 3 Tactics
- 4 Conclusion

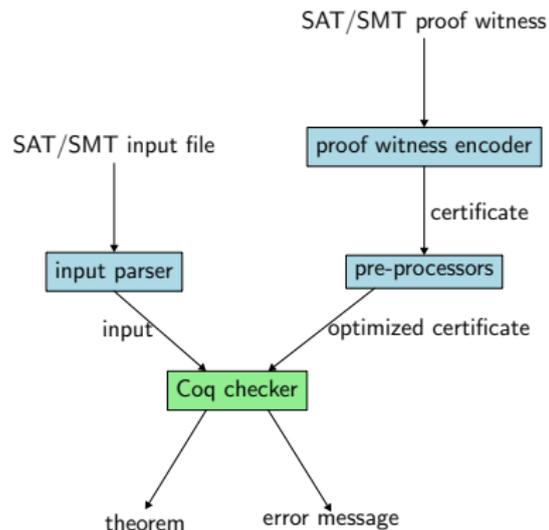
# The heart: a certified checker for **unsatisfiability**



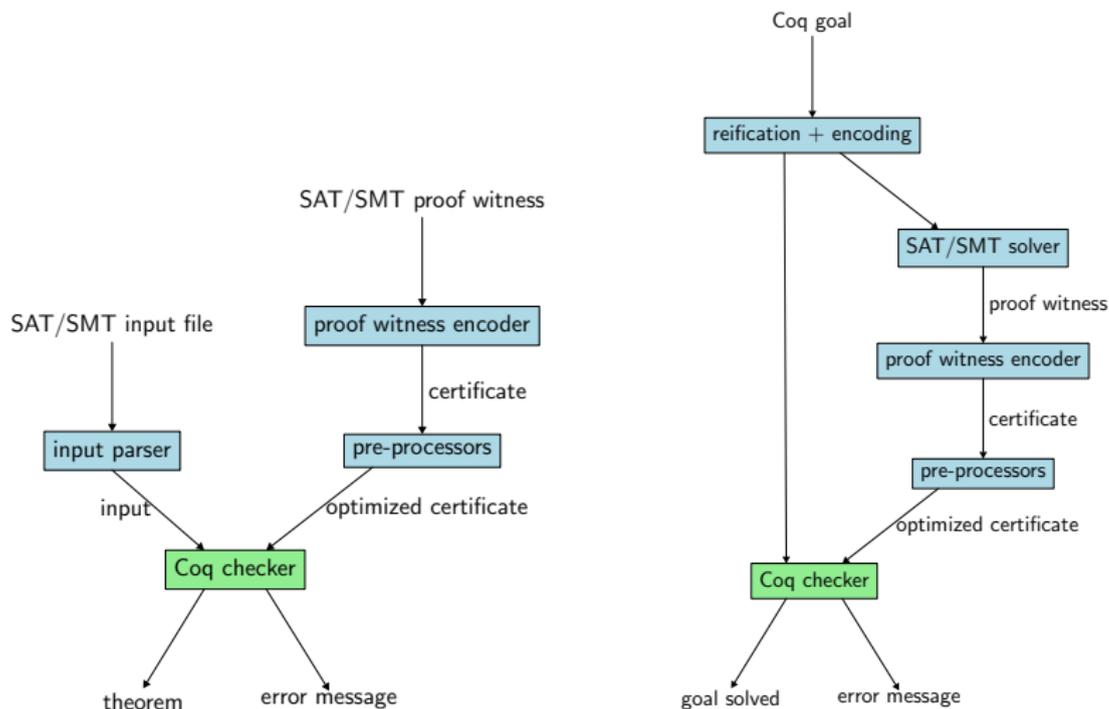
## Certification:

- `checker : formula → certif → bool`
- `correctness :`  
$$\forall \phi c, \text{checker } \phi c = \text{true} \rightarrow \forall \rho, |\phi|_{\rho} = \text{false}$$
- $|\bullet|_{\rho} : \text{formula} \rightarrow \text{bool}$  is an interpretation function
- can be extracted to ML

# The two applications



# The two applications



# Checker input and certificate formats

## Input:

- a first-order formula  $\phi$  in a combination of theories (SMT-LIB2)

## Certificate:

- a proof of the unsatisfiability of  $\phi$  in a combination of theories (in the large sense: modularity)

## Example of a certificate

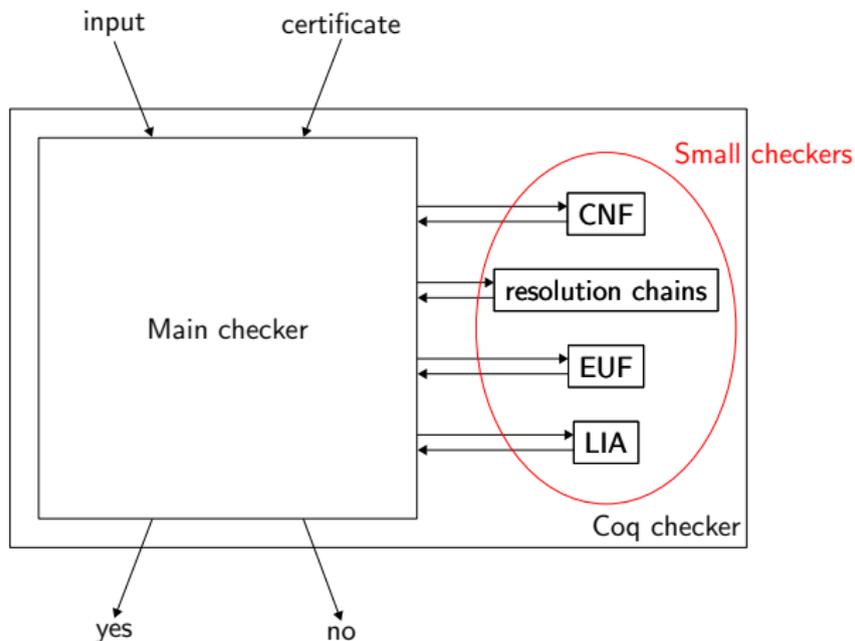
Unsatisfiability of (the conjunction of):  $x \geq 7 \wedge y \leq -4 \quad \neg x \geq 2$

$$\frac{\frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso}}{\square} \text{ Reso}$$

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# A modular checker



# The small checkers and the main checker

A small checker:

- takes some clauses and a piece of certificate as arguments
- returns a clause that is implied

The main checker:

- maintains a set of clauses, initialized with the input
- sequentially shares out each certificate step between the corresponding small checker
- checks that the last obtained clause is the empty clause

The correctness of each small checkers implies the correctness of the whole checker

## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

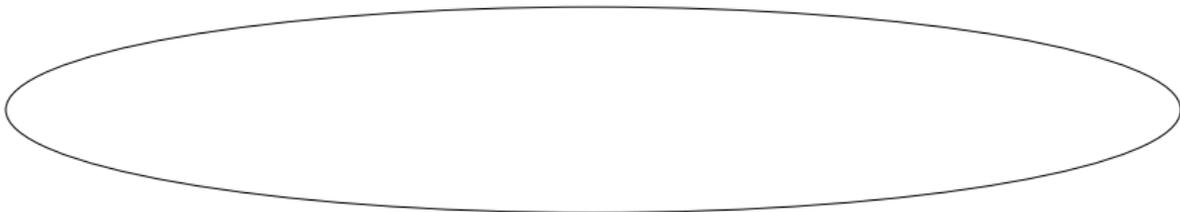
$$\frac{\frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\text{Reso}}}{\text{Reso}} \quad \square$$

## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \qquad \frac{\overline{\neg x \geq 7 \vee x \geq 2} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso} \\
 \hline
 \square \qquad \text{Reso}
 \end{array}$$

A set of clauses:

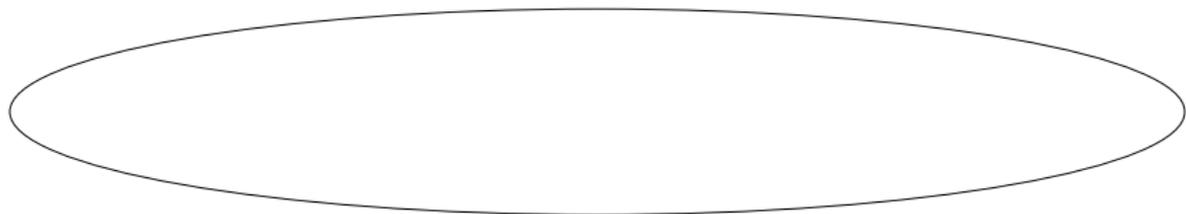


# The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$

$$\frac{\frac{x \geq 7 \wedge y \leq -4}{\text{CNF}} \quad \frac{\text{LIA}}{\neg x \geq 2} \text{ Reso}}{\text{Reso}}$$

A set of clauses:



## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{\text{CNF}} \quad \frac{\text{LIA}}{\text{Reso}} \quad \frac{\neg x \geq 2}{\text{Reso}} \\
 \hline
 \text{Reso}
 \end{array}$$

A set of clauses:

$$x \geq 7 \wedge y \leq -4$$

$$\neg x \geq 2$$

## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

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A set of clauses:

$$x \geq 7 \wedge y \leq -4$$

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A set of clauses:

$$\begin{array}{l} x \geq 7 \wedge y \leq -4 \quad \neg x \geq 7 \vee x \geq 2 \\ \neg x \geq 2 \end{array}$$

## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{\text{CNF}} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\text{LIA}} \quad \neg x \geq 2}{\neg x \geq 7} \text{Reso} \\
 \hline
 \text{Reso}
 \end{array}$$

A set of clauses:

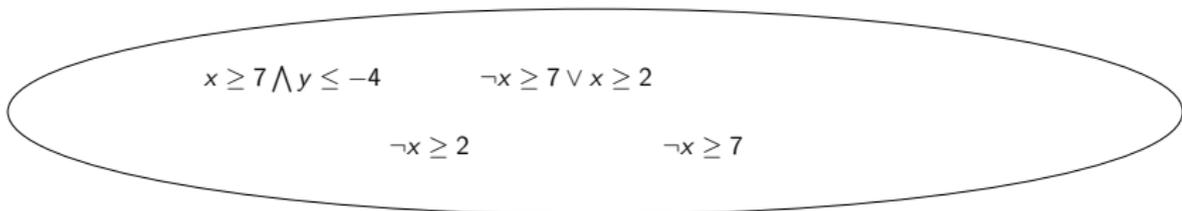
$$\begin{array}{c}
 x \geq 7 \wedge y \leq -4 \quad \neg x \geq 7 \vee x \geq 2 \\
 \neg x \geq 2
 \end{array}$$

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Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

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 \hline
 \text{Reso}
 \end{array}$$

A set of clauses:



## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

$$\frac{\frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\neg x \geq 7 \vee x \geq 2}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso}}{\text{Reso}}$$

A set of clauses:

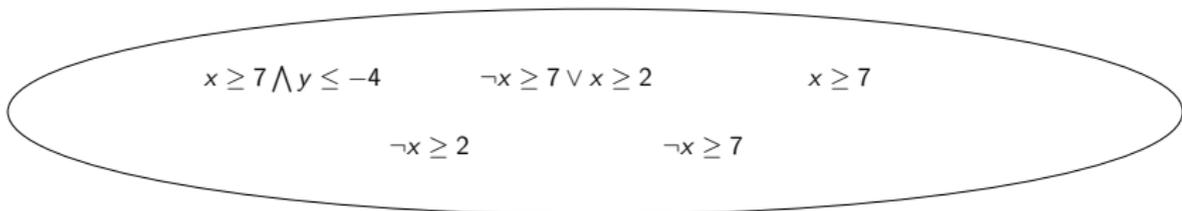
$$\begin{array}{cc} x \geq 7 \wedge y \leq -4 & \neg x \geq 7 \vee x \geq 2 \\ \neg x \geq 2 & \neg x \geq 7 \end{array}$$

## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $\neg x \geq 2$ 

$$\frac{\frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\neg x \geq 7 \vee x \geq 2}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso}}{\text{Reso}}$$

A set of clauses:

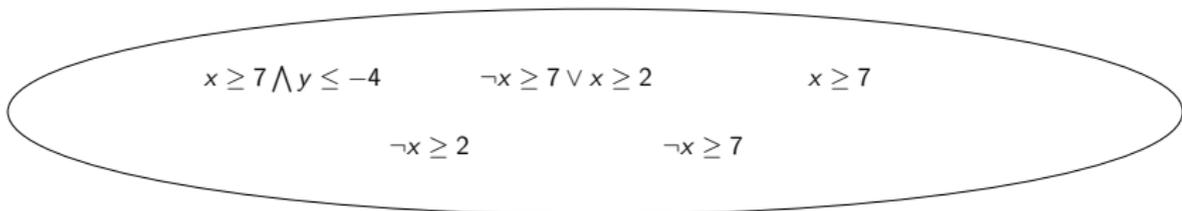


## The main checker by example

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$   $\neg x \geq 2$ 

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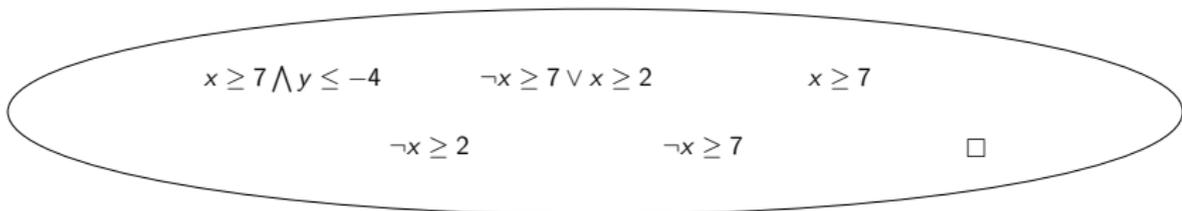


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A set of clauses:

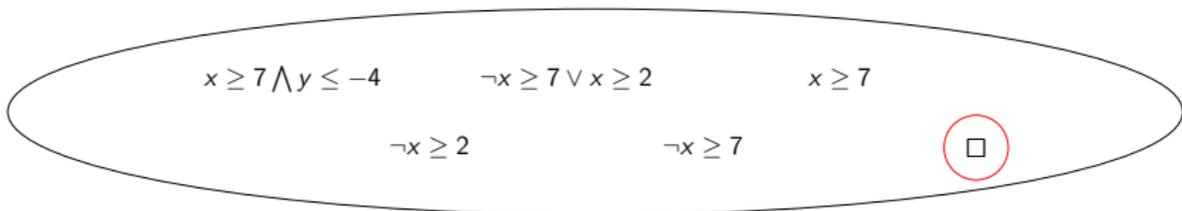


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A set of clauses:



# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 2} \text{ LIA}}{\neg x \geq 7} \text{ Reso} \\
 \hline
 \square \text{ Reso}
 \end{array}$$

# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

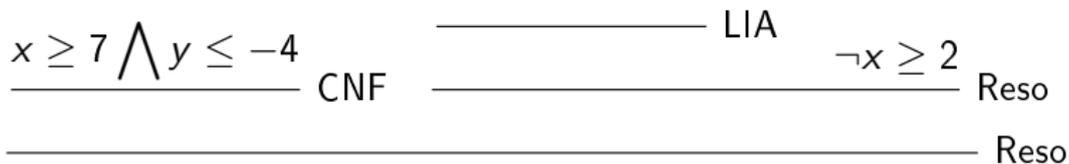
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 \hline
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 \end{array}$$

3 clauses alive at the same time:



# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

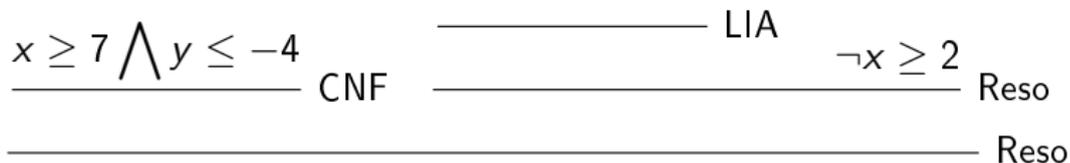


3 clauses alive at the same time:

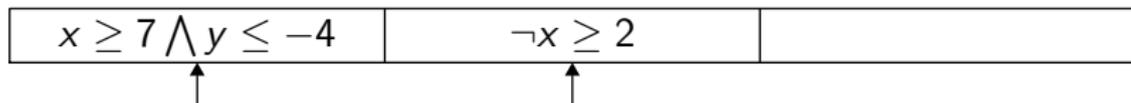


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Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$



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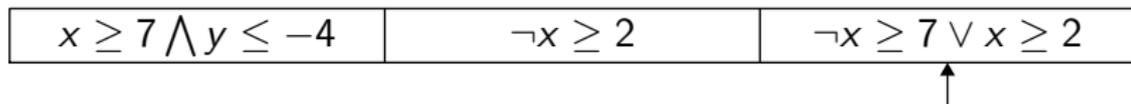
$x \geq 7 \wedge y \leq -4$	$\neg x \geq 2$	
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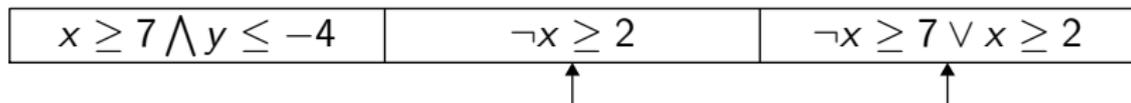
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 \end{array}$$

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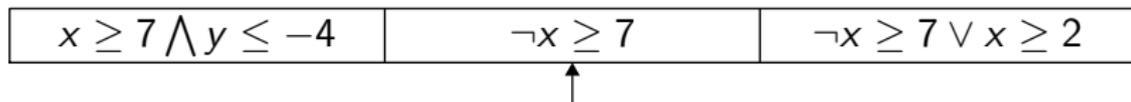


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Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

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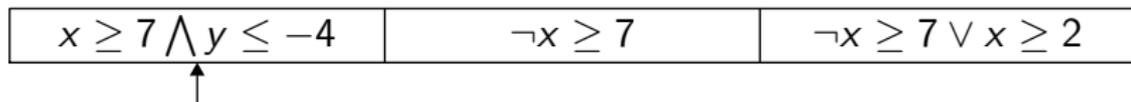
$x \geq 7 \wedge y \leq -4$	$\neg x \geq 7$	$\neg x \geq 7 \vee x \geq 2$
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Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

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 \hline
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 \end{array}$$

3 clauses alive at the same time:

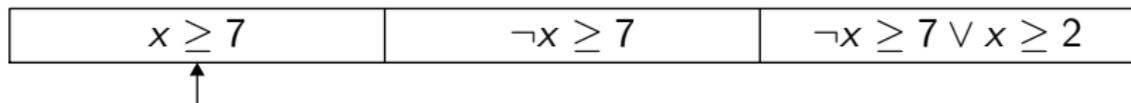


# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4 \quad x < 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso} \\
 \hline
 \text{Reso}
 \end{array}$$

3 clauses alive at the same time:



## Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4 \quad x < 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso} \\
 \hline
 \square \text{ Reso}
 \end{array}$$

3 clauses alive at the same time:

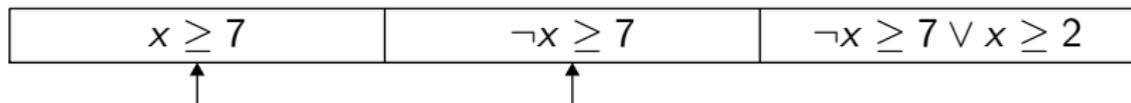
$x \geq 7$	$\neg x \geq 7$	$\neg x \geq 7 \vee x \geq 2$
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# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4 \quad x < 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\overline{\neg x \geq 7 \vee x \geq 2}}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso} \\
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3 clauses alive at the same time:

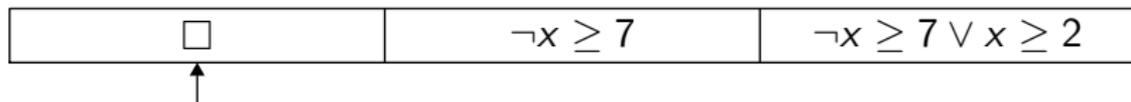


# Example of optimization

Unsatisfiability of:  $x \geq 7 \wedge y \leq -4$        $x < 2$

$$\begin{array}{c}
 \frac{x \geq 7 \wedge y \leq -4}{x \geq 7} \text{ CNF} \quad \frac{\frac{\neg x \geq 7 \vee x \geq 2}{\neg x \geq 7} \text{ LIA} \quad \neg x \geq 2}{\neg x \geq 7} \text{ Reso} \\
 \hline
 \square \quad \text{Reso}
 \end{array}$$

3 clauses alive at the same time:



# Pre-processing

## Other optimizations:

- certificates: cleaning, sharing
- formulas: hash-consing, flattening
- ...

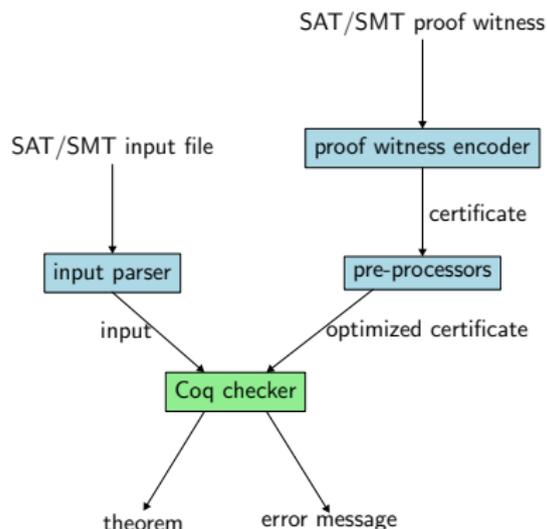
## Proof witness encoding (ZChaff, glucose, veriT, CVC4):

- fit into the format: encoding of nested proofs, DRUP
- recover information: arithmetic, simplifications, quantifiers, holes
- ...

↔ no need to certify: very easily extensible

↔ SMTCoq also safely combines SMT solvers

# Application 1: combinatorial mathematics



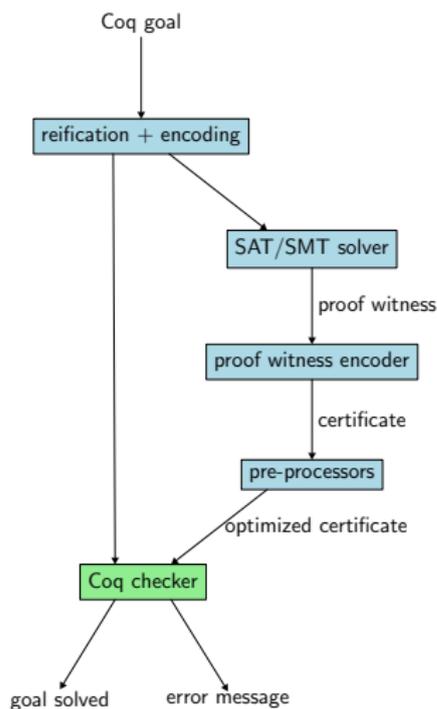
## Efficiency:

- optimization
- native data-structures in Coq (integers, arrays)
- VM and native computation
- for very large witnesses: file reading on the fly

# Outline

- 1 Skeptical interaction with external provers
- 2 Checker
- 3 Tactics
- 4 Conclusion

## Application 2: Coq tactics



### Expressivity:

- CIC  $\rightarrow$  first-order logic
- take advantage of supported theories and quantifier instantiation
- cope with multiple definitions of the same mathematical objects
- deal with classical logic

# Theories currently supported by SMTCoq

## Current small checkers:

- extended resolution
- CNF computation
- equality
- linear integer arithmetic
- bit vectors
- arrays
- quantifiers
- silent simplifications

↔ modularity: they are independent (they only need to agree on formulae) ⇒ easily extensible

# Problem 1: multiple representations of similar objects

## Integers:

- relative integers:  $\mathbb{Z}$ , bigZ
- but also: nat, N, positive, bigN, ...

## Arrays:

- maps, arrays, functions, ...

...and the same for most theories

...and the user may implement its own representation

# A well-known problem

For integers:

- `zify`, `ppsimpl`: does not work in combination with other theories
- *transfer tactics*: requires human effort, works only for isomorphic types

↔ our own conversion tactics, applied before `SMTCoq`

# Example

```

x, y : positive                                f : positive -> positive
=====
((x + 3) = y) -> ((3 < y) /\ ((f (x + 3)) <= (f y)))

```

↓

```

x', y' : Z          Hx' : 0 < x'          Hf'x' : 0 < f' (x' + 3)
f' : Z -> Z        Hy' : 0 < y'          Hf'y' : 0 < f' y'
=====
((x' + 3) = y') -> ((3 < y') /\ (f' (x' + 3) <= f' y'))

```

## Current approach

Ltac code:

- 1 add double conversions at the leaves
- 2 rewrite them bottom-up
- 3 rename to hide remaining conversions

Practical use:

- the user realizes a Coq module
- a functor automatically generates the tactic
- generic approach; implementation given for integers

Perspectives:

- provide implementations for all the theories
- robustness and mix with other SMTCoq features
- investigate a reflexive approach (MetaCoq)

# Classical logic

## Fact:

Classical logic is not needed in general, only in particular use cases that may appear in proofs

## No axiom is added:

- automatic conversion of the quantifier-free part of the goal into a Boolean expression: the user may have to prove decidability of some predicates (or assume it)
- *shallow* treatment of quantifiers

$\Leftrightarrow$  again, a conversion tactic from `Prop` to `bool`, applied before `SMTCoq`

# Quantifiers

Expressivity of SMTCoq:

- goal:  $\overline{\forall \bar{x}. P \bar{x}} \rightarrow \forall \bar{x}. G \bar{x}$
- context:  $\overline{\forall \bar{x}. Q \bar{x}}$

( $\overline{P}$ ,  $\overline{Q}$ ,  $G$  quantifier-free)

In a nutshell:

SMTCoq can instantiate universally-quantified lemmas to prove the goal

# Mixing deep and shallow embeddings in a reflexive tactic

Coq knows how to instantiate universal quantifiers!

- + let's use it
- + no binders in the type of formulae
- + intuitionistic
- a bit restrictive on the expressivity

Deep and shallow small checkers:

- `conj_elim_l` : `formula (* [A ∧ B] *) → formula (* [A] *)`
- `forall_inst` : `(p:Prop) (* forall x. P x *) → (h:p) → formula (* [P a] *)`

↪ an original way to write reflexive tactics

# To obtain full first-order logic

Skolemization:

- need to carefully exhibit “classical” requirements

but in most use cases, lemma instantiation is sufficient

# Perspectives of the tactics

## Expressivity:

- inductive types and predicates (support from SMT solvers)
- dependent types (encoding)
- ...

## Robustness

## Benchmarks:

- CompCert
- SSReflect and Mathematical Components

# Outline

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# Play with it!

Combinatorial problems? Tedious Coq proofs?

`smtcoq.github.io`

User experience (and participation) welcome!