Formal verification of a static analyzer: abstract interpretation in type theory

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In memoriam Radhia Cousot, † 2014

With thanks to. . .

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Plan

1. An overview of static analysis
2. Abstract interpretation, in set theory and in type theory
3. Scaling up: the Verasco project
4. Conclusions and future work

Static analysis in a nutshell

Statically infer properties of a program that hold for all its executions.

At this program point, $0 < x \leq y$ *and pointer* p *is not* `NULL`.

Emphasis on infer: no help from the programmer.
(E.g. loop invariants are not written in the source.)

Emphasis on statically:

- The inputs to the program are not known.
- The analysis must terminate.
- The analysis must run in reasonable time and space.

Example of properties that can be inferred

Properties of the value of one variable: (value analysis)

$x = a$	constant propagation
$x > 0$ ou $x = 0$ ou $x < 0$	signs
$x \in [a, b]$	intervalles
$x = a \pmod{b}$	congruences
$\texttt{valid}(p[a \ldots b])$	memory validity
p `pointsTo` x or $p \neq q$	(non-) aliasing between pointers

(a, b, c are constants inferred by the analyzer.)

Example of properties that can be inferred

Properties of several variables: (relational analysis)

$\sum a_i x_i \leq c$	polyhedra
$\pm x_1 \pm \cdots \pm x_n \leq c$	octogons
$expr_1 = expr_2$	Herbrand equivalences
$\textit{doubly-linked-list}(p)$	shape analysis

Non-functional properties:

- Memory consumption.
- Worst-case execution time (WCET).

Using static analysis for code optimization

Apply algebraic identities when their conditions are met:

`x / 4`	$\to$	`x >> 2`	if analysis says $\mathtt{x} \geq 0$
`x + 1`	$\to$	`1`	if analysis says $\mathtt{x} = 0$

Optimize array accesses and pointer dereferences:

`a[i]=1; a[j]=2; x=a[i];`	$\to$	`a[i]=1; a[j]=2; x=1;` if analysis says $\mathtt{i} \neq \mathtt{j}$
`*p = a; x = *q;`	$\to$	`x = *q; *p = a;` if analysis says $\mathtt{p} \neq \mathtt{q}$

Automatic parallelization:

$$loop_1; loop_2 \to loop_1 \parallel loop_2 \quad \text{if } polyh(loop_1) \cap polyh(loop_2) = \emptyset$$

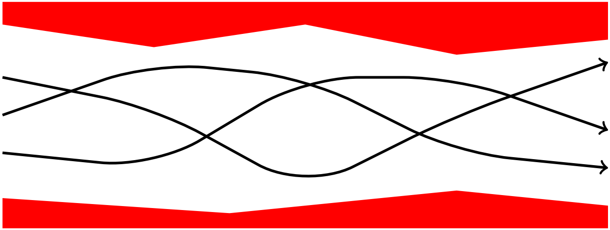
Using static analysis for verification

Use the results of static analysis to prove the absence of certain run-time errors:

$$x \in [a, b] \wedge 0 \notin [a, b] \implies x/y \text{ cannot fail}$$

$$\texttt{valid}(p[a \ldots b]) \wedge i \in [a, b] \implies \texttt{p[i]} \text{ cannot fail}$$

Report an alarm otherwise.



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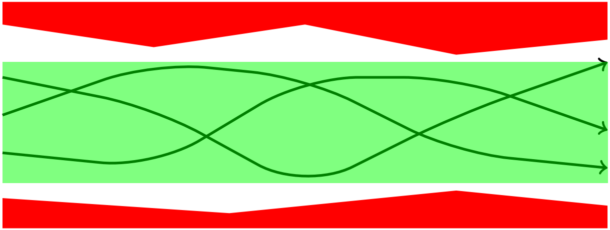
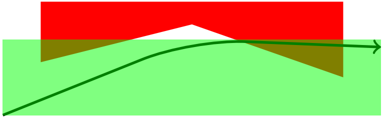
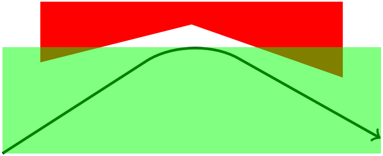


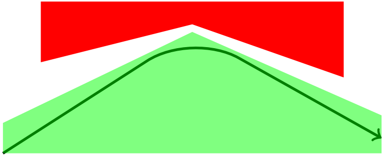
True alarms, false alarms



True alarm
(wrong behavior)



False alarm
(analysis too imprecise)



More precise analysis (polyhedron instead of intervals):
the false alarm goes away.

Some properties verifiable by static analysis

Absence of run-time errors:

- Arrays and pointers:
  - No out-of-bound accesses.
  - No dereferencing the null pointer.
  - No access after a `free`.
  - Alignment constraints are respected.
- Integer arithmetic:
  - No division by zero.
  - No (signed) arithmetic overflows.
- Floating-point arithmetic:
  - No arithmetic overflows (result is $\pm\infty$)
  - No undefined operations (result *Not a Number*)
  - No catastrophic cancellation.

Simple programmer-inserted assertions:

e.g. `assert (0 <= x && x < sizeof(tbl))`.

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Basic idea:
analyzing a program is
executing it with a nonstandard semantics

Abstract interpretation in a nutshell

Execute ("interpret") the program with a semantics that:

- Computes over an abstract domain of the desired properties (e.g. "$x \in [a, b]$" for interval analysis) instead of computing with concrete values and states (e.g. numbers).
- Handle Boolean conditions even if they cannot be resolved statically:
  - The `then` and `else` branches of an `if` are both taken $\rightarrow$ joins.
  - Loops and recursions execute arbitrarily many times $\rightarrow$ fixpoints.
- Always terminates.

Examples of abstract interpretation

In the concrete	In the abstract
$\{\ x = 3, y = 1\ \}$	$\{\ x^\# = [0, 9], y^\# = [-1, 1]\ \}$

```
z = x + 2 * y;
```

$\{\ z = 3 + 2 \times 1 = 5\ \}$	$\{\ z^\# = [0, 9] +^\# 2 \times^\# [-1, 1] = [-2, 11]\ \}$

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z = x + 2 * y;
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$\{\ z = 3 + 2 \times 1 = 5\ \}$	$\{\ z^\# = [0,9] +^\# 2 \times^\# [-1,1] = [-2,11]\ \}$
$\{\ b = \mathtt{true}, x = 3, y = 1\ \}$	$\{\ b^\# = \top, x^\# = [0,9], y^\# = [-1,1]\ \}$

```
z = (if b then x else y);
```

$\{\ z = 3\ \}$	$\{\ z^\# = [0,9] \sqcup [-1,1] = [-1,9]\ \}$

Idea #2:
a variable can have different abstractions
at different program points

Sensitivity to control flow

Imperative variable assignment:

$\{\ x^{\#} = [0,9]\ \}$

```
x = x + 1;
```

$\{\ x^{\#} = [1,10]\ \}$

Refining the abstraction at conditionals:

$\{\ x^{\#} = [0,9]\ \}$

```
if (x == 0) {
```

$\{\ x^{\#} = [0,0]\ \}$

```
    ...
} else {
```

$\{\ x^{\#} = [1,9]\ \}$

```
    ...
}
```

Sensitivity to control flow

Contrast with dependent pattern-matching, where the type of the scrutinee is unchanged, but additional facts are added to the environment.

```
match eq_dec x 0 with
| left  (EQ: x = 0)   => ...
| right (NEQ: x <> 0) => ...
end.

match x as z return x = z -> T with
| None   => fun (P: x = None)   => ...
| Some y => fun (P: x = Some y) => ...
end (refl_equal x).
```

Idea #3:
we can also infer relations
between the values of several variables

Non-relational / relational analysis

Non-relational analysis:

$$\textit{abstract environment} = \textit{variable} \mapsto \textit{abstract value}$$

(Like simple typing environments.)

Relational analysis:

abstract environments are a domain of their own, featuring:

- a semi-lattice structure: $\bot$, $\top$, $\sqsubseteq$, $\sqcup$
- an abstract operation for assignment / binding.

Example: polyhedra, i.e. conjunctions of linear inequalities $\sum a_i x_i \leq c$.

Idea # 4: widening
fixpoints can be computed
even in non-well-founded domains

Fixpoints – the recurring problem

Static analysis of a loop:

```
while (...) {        { e# = X0 }
                     { e# = X }
    ...
                     { e# = Φ(X) }
}
```

$\{ \; e^{\#} = X_0 \; \}$

$\{ \; e^{\#} = X \; \}$

$\{ \; e^{\#} = \Phi(X) \; \}$

Given X_0 (the abstract state before the loop)
and Φ (the transfer function for the loop body),
find X (the loop invariant).

$X \sqsupseteq X_0$ (first iteration) $\qquad X \sqsupseteq \Phi(X)$ (next iterations)

X is, ideally, the smallest fixpoint of $F = X \mapsto X_0 \sqcup \Phi(X)$
or at least any post-fixpoint of $F \qquad (X \sqsupseteq F(X))$.

Paradise

Theorem (Tarski)

Let $(A, \sqsubseteq, \bot)$ a partially ordered set such that $\sqsupset$ is well founded (no infinite increasing sequences).
Let $F : A \rightarrow A$ an increasing function.
Then F has a smallest fixpoint, obtained by finite iteration from $\bot$:

$$\exists n, \quad \bot \sqsubset F(\bot) \sqsubset \ldots \sqsubset F^n(\bot) = F^{n+1}(\bot)$$

Paradise lost

Most abstract domains are not well founded. Examples:

- Integer intervals: $[0,0] \sqsubset [0,1] \sqsubset [0,2] \sqsubset \cdots \sqsubset [0,n] \sqsubset \cdots$
- Environments: *variable* $\mapsto$ *abstract values*.

Moreover, even when Tarski iteration converges, it converges too slowly:

```
x = 0;  while (x <= 10000) { x = x + 1; }
```

(Starting with $x^\# = [0,0]$, it takes 10000 iterations to reach the fixpoint $x^\# = [0,10000]$.)

Paradise regained: widening

A widening operator $\nabla : A \to A \to A$ computes a majorant of its second argument in such a way that the following iteration converges always and quickly:

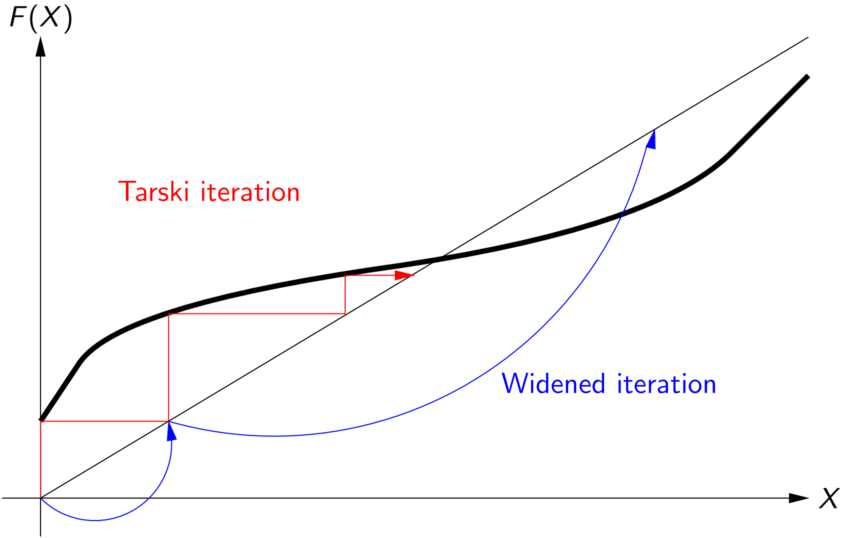
$$X_0 = \bot \qquad X_{i+1} = \begin{cases} X_i & \text{if } F(X_i) \sqsubseteq X_i \\ X_i \nabla F(X_i) & \text{otherwise} \end{cases}$$

The limit X of this sequence is a post-fixpoint: $F(X) \sqsubseteq X$.

Example: widening for intervals:

$$[l_1, u_1] \nabla [l_2, u_2] = \begin{array}{l} [\texttt{if } l_2 < l_1 \texttt{ then } -\infty \texttt{ else } l_1, \\ \ \texttt{if } u_2 > u_1 \texttt{ then } \infty \texttt{ else } u_1] \end{array}$$

Widening in action



Narrowing the post-fixpoint

The quality of the post-fixpoint can be improved by iterating F some more:

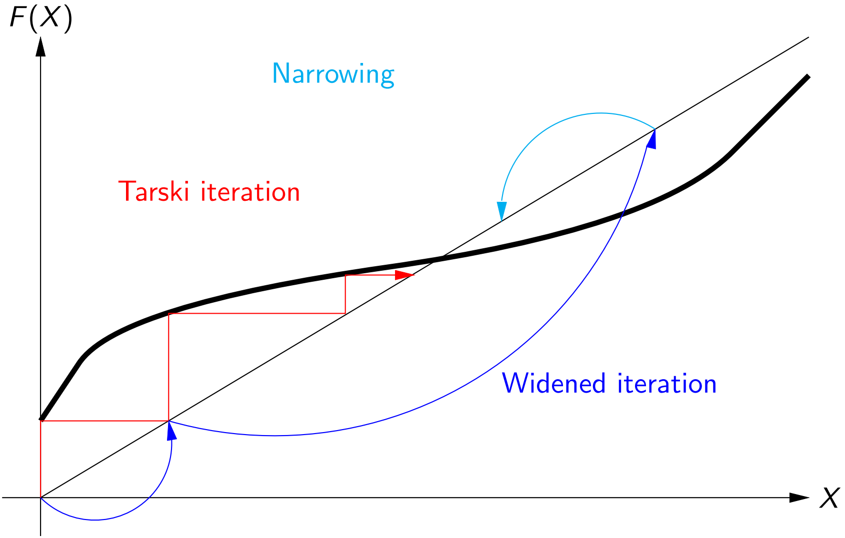
$$Y_0 = \text{a post-fixpoint} \qquad Y_{i+1} = F(Y_i)$$

If F is increasing, each Y_i is a post-fixpoint: $F(Y_i) \sqsubseteq Y_i$.

Often, $Y_i \sqsubset Y_0$, improving the analysis quality.

Iteration can be stopped when Y_i is a fixpoint, or at any time.

Widening plus narrowing in action



Specification of widening

A simple variation on the constructive definition of well foundedness:

```
Inductive Acc: A -> Prop :=
| Acc_intro: ∀x,
    (∀y, y⊐x -> Acc y) ->
    Acc x.

Definition well_founded :=
  ∀x, Acc x.
```

```
Inductive AccW: A -> Prop :=
| AccW_intro: ∀x,
    (∀y, y⊐x -> AccW (x∇y)) ->
    AccW x.

Definition widening_correct :=
  ∀x, AccW x.
```

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Definition well_founded :=
  ∀x, Acc x.
```

```
Definition widening_correct :=
  ∀x, AccW x.
```

Even Coq understands that widened iteration terminates:

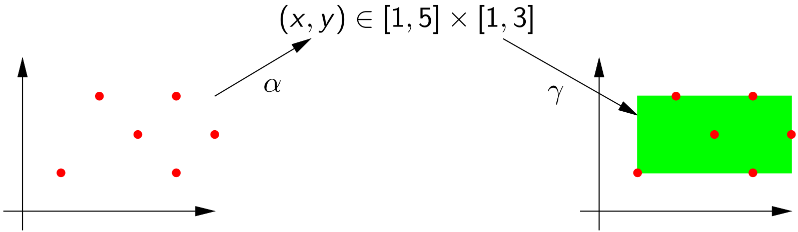
```
Fixpoint postfixpoint (F: A->A) (x: A) (acc: AccW x) {struct acc} :=
  let y := F x in
  match decide (x⊑y) with
  | left  LE => x
  | right GT => postfixpoint F (x∇y) (AccW_inv x acc y GT)
  end.
```

Idea #6: Galois connections:
abstract operators can be calculated
in a systematic, sound, and optimal manner

A Galois connection

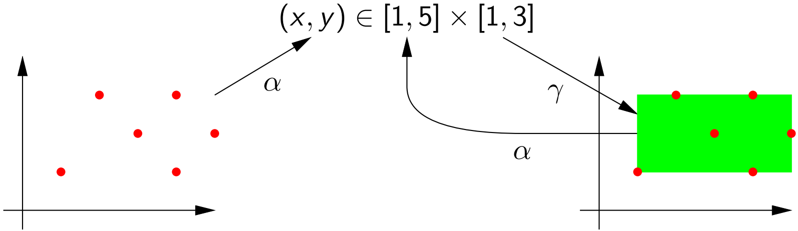
A semi-lattice $\mathcal{A}, \sqsubseteq$ of abstract states and two functions:

- Abstraction function α : set of concrete states $\rightarrow$ abstract state
- Concretization function γ : abstract state $\rightarrow$ set of concrete states



E.g. for intervals $\alpha(S) = [\inf S, \sup S]$ and $\gamma([a, b]) = \{x \mid a \leq x \leq b\}$.

Axioms of Galois connections



The adjunction property:

$$\forall a, S, \quad \alpha(S) \sqsubset a \Leftrightarrow S \subseteq \gamma(a)$$

or, equivalently:

$$\begin{array}{ll} & \alpha \text{ increasing} \\ \wedge & \gamma \text{ increasing} \\ \wedge & \forall S, \quad S \subseteq \gamma(\alpha(S)) \quad \text{(soundness)} \\ \wedge & \forall a, \quad \alpha(\gamma(a)) \sqsubseteq a \quad \text{(optimality)} \end{array}$$

Calculating abstract operators

For any concrete operator $F : C \to C$ we define its abstraction $F^\# : A \to A$ by

$$F^\#(a) = \alpha\{F(x) \mid x \in \gamma(a)\}$$

This abstract operator is:

- Sound: if $x \in \gamma(a)$ then $F(x) \in \gamma(F^\#(a))$.
- Optimally precise: every a' such that $x \in \gamma(a) \Rightarrow F(x) \in \gamma(a')$ is such that $F^\#(a) \sqsubseteq a'$.

Moreover, an algorithmic definition of $F^\#$ can be calculated from the definition above.

Calculating $+^\#$ for intervals

$$
\begin{aligned}
& [a_1, b_1] +^\# [a_2, b_2] \\
= \quad & \alpha\{x_1 + x_2 \mid x_1 \in \gamma[a_1, b_1], x_2 \in \gamma[a_2, b_2]\} \\
= \quad & [\ \inf\{x_1 + x_2 \mid a_1 \le x_1 \le b_1, a_2 \le x_2 \le b_2\}, \\
& \quad \sup\{x_1 + x_2 \mid a_1 \le x_1 \le b_1, a_2 \le x_2 \le b_2\}\ ] \\
= \quad & [+\infty, -\infty] \text{ if } a_1 > b_1 \text{ or } a_2 > b_2 \\
= \quad & [a_1 + b_1, a_2 + b_2] \text{ otherwise}
\end{aligned}
$$

Note: the intuitive definition $[a_1, b_1] +^\# [a_2, b_2] = [a_1 + b_1, a_2 + b_2]$ is sound but not optimal.

Trouble ahead:
Galois connections in type theory

Type-theoretic difficulties

Minor issue: the calculations of abstract operators are poorly supported by interactive theorem provers such as Coq:

$$F^{\#} a \;=\; \alpha(\lambda x. P) \;\underset{\substack{\uparrow \\ \text{because } \forall x, P \Leftrightarrow P'}}{=}\; \alpha(\lambda x. P') \;=\; \ldots$$

Either:

- use setoid equalities everywhere, or
- add extensionality axioms (functional, propositional).

Type-theoretic difficulties

Major issue: γ is easily modeled as

$$\gamma : A \rightarrow (C \rightarrow \texttt{Prop}) \quad \text{(two-place predicate)}$$

but α is generally not computable as soon as C is infinite:

$\alpha : (C \rightarrow \texttt{Prop}) \rightarrow A$ morally constant functions only?
$\alpha : (C \rightarrow \texttt{bool}) \rightarrow A$ can only query a finite number of C's

(E.g. $\alpha(S) = [\inf S, \sup S]$, no more computable than inf and sup.)

$\rightarrow$ Need more axioms (description, Hilbert's epsilon).

Fundamental difficulty

For some domains, the abstraction function α does not exist!
(The optimality condition $a \sqsubseteq \alpha(\gamma(a))$ cannot be satisfied.)

Example 1: intervals of rationals.

$$\alpha\{x \mid x^2 \leq 2\} = ???$$

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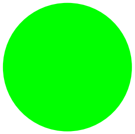
Example 2: polyhedra

$$\alpha\{(x, y) \mid x^2 + y^2 \leq 1\} = ???$$



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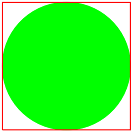

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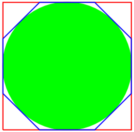

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Plan B:
soundness (γ) is essential,
optimality (α) is optional

Getting rid of α

Remember the two properties of abstract operators $F^{\#}$ calculated from $F^{\#}(a) = \alpha\{F(x) \mid x \in \gamma(a)\}$:

1. Soundness: if $x \in \gamma(a)$ then $F(x) \in \gamma(F^{\#}(a))$.
2. Optimality: every a' such that $x \in \gamma(a) \Rightarrow F(x) \in \gamma(a')$ is such that $F^{\#}(a) \sqsubseteq a'$.

Instead of calculating $F^{\#}$, we can guess a definition for $F^{\#}$, then verify

- property 1: soundness (mandatory!)
- possibly property 2: optimality (optional sanity check).

These proofs only need the concretization relation γ, which is unproblematic.

Soundness first!

Having made optimality entirely optional, we can further simplify the analyzer and its soundness proof, while increasing its algorithmic efficiency:

- Abstract operators that return over-approximations (or just $\top$) in difficult / costly cases.
- Join operators $\sqcup$ that return an upper bound for their arguments but not necessarily the least upper bound.
- "Fixpoint" iterations that return a post-fixpoint but not necessarily the smallest (widening + return $\top$ when running out of fuel).
- Validation a posteriori of algorithmically-complex operations, performed by an untrusted external oracle. (Next slide.)

Validation a posteriori

Some abstract operations can be implemented by unverified code if it is easy to validate the results a posteriori by a validator. Only the validator needs to be proved correct.

Example: the join operator $\sqcup$ over polyhedra.

Computing the join (convex hull) vs. Inclusion test (Presburger formula)

The inclusion test can itself use validation a posteriori.
(Cf. talk by Fouilhe, Boulmé and Périn.)

Plan

The Verasco project

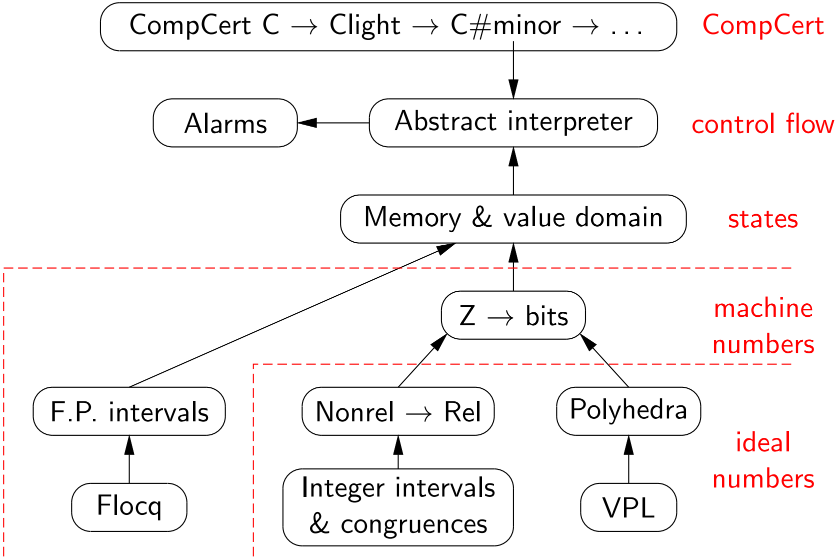
Inria Celtique, Gallium, Abstraction, Toccata + Verimag + Airbus

Goal: develop and verify in Coq a realistic static analyzer by abstract interpretation:

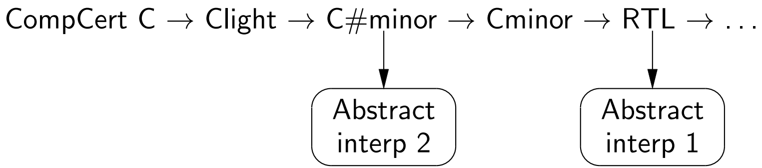
- Language analyzed: the CompCert subset of C.
- Nontrivial abstract domains, including relational domains.
- Modular architecture inspired from Astrée's.
- Decent alarm reporting.

Slogan: if "CompCert = 1/10th of GCC but formally verified", likewise "Verasco = 1/10th of Astrée but formally verified".

Architecture



Upper layer: the abstract interpreter



Connected to the intermediate languages of the CompCert compiler.

Parameterized by a relational abstract domain for execution states (environment + memory state + call stack).

1. Abstract interpreter for RTL (Blazy, Maronèze, Pichardie, SAS 2013)
   Unstructured control → per-function fixpoints (Bourdoncle).
2. Abstract interpreter for C#minor (Jourdan, in progress)
   Local fixpoints for each loop + per-function fixpoint for `goto` + per-program fixpoint for function calls.

Lower layer: numerical domains

Non-relational:

- Integer intervals and congruences (over **Z**).
- Floating-point intervals (on top of the Flocq library).

Relational:

- The VPL library (Fouilhé, Monniaux, Périn, SAS 2013): polyhedra with rational coefficients, implemented in OCaml, producing certificates verifiable in Coq.
- Integration in progress in Verasco.

What is a generic interface for a numerical domain?

For a non-relational domain:

- A semilattice $(A, \sqsubseteq)$ of abstract values.
- A concretization relation $\gamma : A \to \mathbf{Z} \to \texttt{Prop}$
- Abstract operators such as

```
add: A -> A -> A;
add_sound: forall a b x y,
    x ∈ γ a -> y ∈ γ b -> (x + y) ∈ γ (add a b);
```

- Inverse abstract operators (to refine abstractions based on the results of conditionals) such as

```
eq_inv: A -> A -> bool -> A * A;
eq_inv_sound: forall a b c x y,
    x ∈ γ a -> y ∈ γ b ->
    (if c then x = y else x <> y) ->
    x ∈ γ (fst (eq_inv a b c))
    ∧ y ∈ γ (snd (eq_inv a b c));
```

What is a generic interface for a numerical domain?

For a relational domain, the main abstract operations are:

- `assign` *var* = *expr*
- `forget` *var* = *any-value*
- `assume` *expr* is true or *expr* is false

var are program variables or abstract memory locations.

expr are simple expressions ($+ \ - \ \times$ `div` `mod` ...) over variables and constants.

To report alarms, we also need to query the domain, e.g. "is $\mathtt{x} < \mathtt{y}$?" or "is $\mathtt{x} \ \mathtt{mod} \ 4 = 0$?". The basic query is

- `get_itv` *expr* $\rightarrow$ *variation interval*

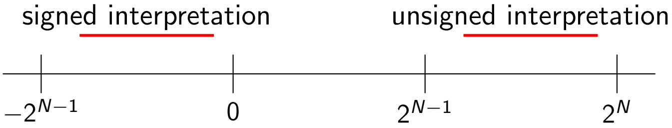
(Next slide: Coq interface.)

```
Class ab_ideal_env (var t:Type) `{EqDec var}: Type := {
  id_wl:> weak_lattice t;
  id_gamma:> gamma_op t (var->ideal_num);
  id_adom:> adom t (var->ideal_num) id_wl id_gamma;
  get_itv: iexpr var -> t -> IdealIntervals.abs+⊥;
  assign: var -> iexpr var -> t -> t+⊥;
  forget: var -> t -> t+⊥;
  assume: iexpr var -> bool -> t -> t+⊥;
  get_itv_sound: forall e ρ ab,
    ρ ∈ γ ab ->
    eval_iexpr ρ e ⊆ γ (get_itv e ab);
  assign_sound: forall x e ρ n ab,
    ρ ∈ γ ab ->
    n ∈ eval_iexpr ρ e ->
    (upd ρ x n) ∈ γ (assign x e ab);
  forget_sound: forall x ρ n ab,
    ρ ∈ γ ab ->
    (upd ρ x n) ∈ γ (forget x ab);
  assume_sound: forall c ρ ab b,
    ρ ∈ γ ab ->
    (INz (if b:bool then 1 else 0)) ∈ eval_iexpr ρ c ->
    ρ ∈ γ (assume c b ab)
}.
```

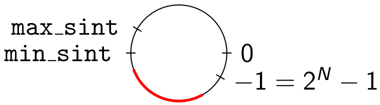
Machine integers vs. mathematical integers

Machine integers = N-bit vectors, with arithmetic modulo 2^N, and two possible interpretations (signed or unsigned).

For intervals, ad-hoc solutions based on pairs of **Z**-intervals:



or on cyclic intervals:



What about relational domains?

A domain transformer for machine integers

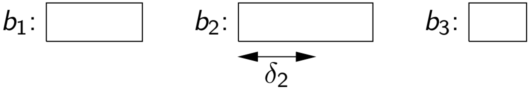
(J-H. Jourdan)

Given a relational domain (A, γ) over $\mathbf{Z}$, construct a relational domain over N-bit machine integers as follows:

- Same abstract domain A.
- New concretization:
  $\gamma'(a) = \{b : \mathit{bitvect}(N) \mid \exists n : \mathbf{Z}, n \in \gamma(a) \land n = b \pmod{2^N}\}$
- Same abstract operators for addition, subtraction, multiplication.
- For other operators (comparisons, division, ...): try first to reduce the ideal integers modulo 2^N to the interval $[0, 2^N)$ or $[-2^{N-1}, 2^{N-1})$, depending on whether the operation is signed or unsigned.

Middle layer: abstracting memory and state

The CompCert memory model: memory location = block b $\times$ offset δ.



Abstraction of offsets $\rightarrow$ integer domain.

Abstraction of blocks:

- First attempt (Pichardie): 1 concrete block = 1 abstract block "global variable x" or "local variable y of function f".
- Recursion, dynamic allocation $\rightarrow$ need for imprecise abstract blocks (standing for several concrete blocks).
- In progress (Laporte): abstract memory model with block fusion and weak updates.

Plan

Conclusions

Trying to bridge elegant foundations and nitty-gritty details
(low-level language, algorithmic efficiency).

Abstract interpretation is a very effective guideline once we forget about optimality of the analysis.

Future work

Much remains to be done to reach a realistic static analyzer:

- "Good" abstractions for memory.
- More (combinations of) abstract domains:
  symbolic equalities, reduced products, trace partitioning, . . .
- Algorithmic efficiency needs more work, esp. on sharing between representations of abstract states.
- Good alarm reports.
- Debugging the precision of the analyses.

One step at a time. . .

. . . we get closer to the formal verification of the tools that participate in the production and verification of critical embedded software.