

# Visitors Unchained

Using **visitors**  
to traverse **abstract syntax with binding**

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May 22, 2017

# Visitors



## Installation & configuration

Installation:

```
opam update
opam install visitors
```

To configure ocamlbuild, add this in `_tags`:

```
true: \
  package(visitors.ppx), \
  package(visitors.runtime)
```

To configure Merlin, add this in `.merlin`:

```
PKG visitors.ppx
PKG visitors.runtime
```

## An “iter” visitor

Annotating a type definition with `[@@deriving visitors { ... }]`...

```
type expr =  
  | EConst of int  
  | EAdd of expr * expr  
  [@@deriving visitors { variety = "iter" }]
```

## An “iter” visitor

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type expr =  
  | EConst of int  
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  [@@deriving visitors { variety = "iter" }]
```

```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    ()  
  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```

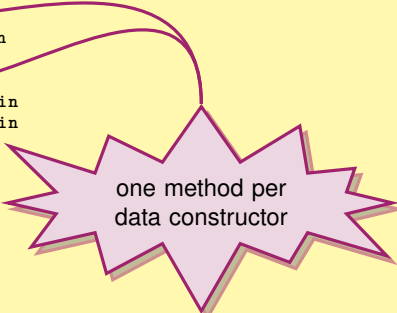
... causes a **visitor class** to be auto-generated.

## An “iter” visitor

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  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



one method per  
data constructor


... causes a **visitor class** to be auto-generated.

## An “iter” visitor

Annotating a type definition with `[@@deriving visitors { ... }]`...

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```
class virtual ['self] iter = object (self : 'self)  
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    let r0 = self#visit_int env c0 in  
    ()  
  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



one method per  
data type

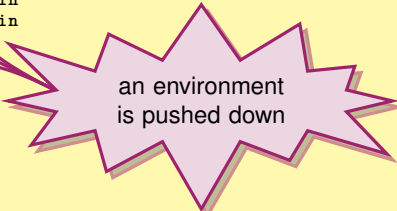
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class virtual ['self] iter = object (self : 'self)  
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  method visit_EConst env c0 =  
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    ()  
  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



an environment  
is pushed down

... causes a **visitor class** to be auto-generated.

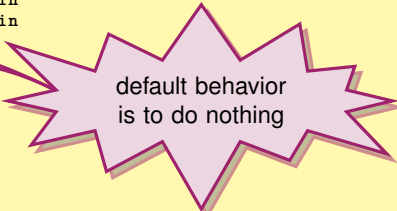


## An “iter” visitor

Annotating a type definition with `[@@deriving visitors { ... }]`...

```
type expr =  
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```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
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    ()  
  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



default behavior  
is to do nothing

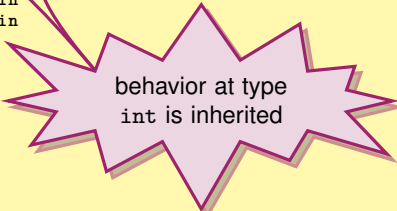
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## An “iter” visitor

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```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    ()  
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    let r1 = self#visit_expr env c1 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



behavior at type  
int is inherited

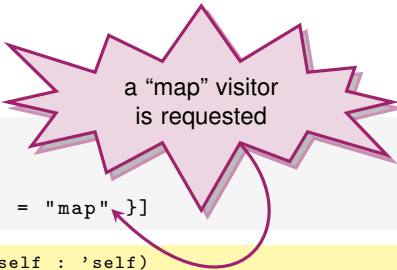
... causes a **visitor class** to be auto-generated.

## A “map” visitor

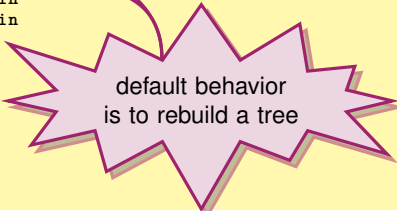
There are several varieties of visitors:

```
type expr =  
  | EConst of int  
  | EAdd of expr * expr  
  [@@deriving visitors { variety = "map" }]
```

```
class virtual ['self] map = object (self : 'self)  
  inherit [_] VisitorsRuntime.map  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    EConst r0  
  method visit_EAdd env c0 c1 =  
    let r0 = self#visit_expr env c0 in  
    let r1 = self#visit_expr env c1 in  
    EAdd (r0, r1)  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



a “map” visitor  
is requested



default behavior  
is to rebuild a tree

## Using a “map” visitor

**Inherit** a visitor class and **override** one or more methods:

```
let add e1 e2 =      (* A smart constructor. *)
  match e1, e2 with
  | EConst 0, e
  | e, EConst 0 -> e
  | _, _ ->         EAdd (e1, e2)

let optimize : expr -> expr =
  let v = object (self)
    inherit [_] map
    method! visit_EAdd env e1 e2 =
      add
        (self#visit_expr env e1)
        (self#visit_expr env e2)
  end in
  v # visit_expr ()
```

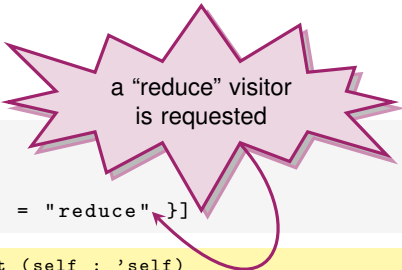
This addition-optimization pass is **unchanged** if more expression forms are added.

## A “reduce” visitor

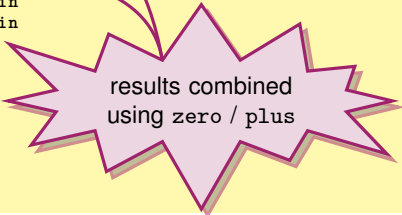
Here is another variety:

```
type expr =  
  | EConst of (int[@opaque])  
  | EAdd of expr * expr  
  [@@deriving visitors { variety = "reduce" }]
```

```
class virtual ['self] reduce = object (self : 'self)  
  inherit [_] VisitorsRuntime.reduce  
  method visit_EConst env c0 =  
    let s0 = (fun this -> self#zero) c0 in  
    s0  
  method visit_EAdd env c0 c1 =  
    let s0 = self#visit_expr env c0 in  
    let s1 = self#visit_expr env c1 in  
    self#plus s0 s1  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



a “reduce” visitor  
is requested



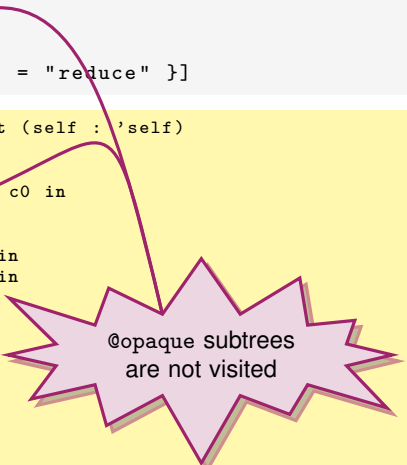
results combined  
using zero / plus

## A “reduce” visitor

Here is another variety:

```
type expr =  
  | EConst of (int[@opaque])  
  | EAdd of expr * expr  
  [@@deriving visitors { variety = "reduce" }]
```

```
class virtual ['self] reduce = object (self : 'self)  
  inherit [_] VisitorsRuntime.reduce  
  method visit_EConst env c0 =  
    let s0 = (fun this -> self#zero) c0 in  
    s0  
  method visit_EAdd env c0 c1 =  
    let s0 = self#visit_expr env c0 in  
    let s1 = self#visit_expr env c1 in  
    self#plus s0 s1  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



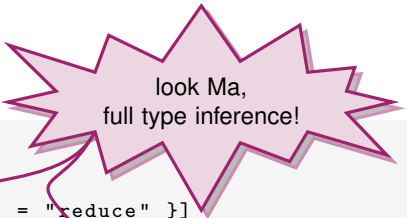
@opaque subtrees  
are not visited

## A “reduce” visitor

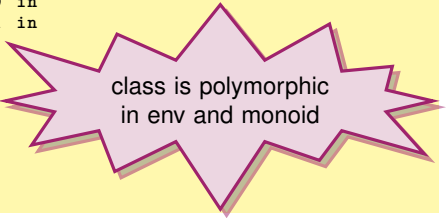
Here is another variety:

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type expr =  
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```

```
class virtual ['self] reduce = object (self : 'self)  
  inherit [_] VisitorsRuntime.reduce  
  method visit_EConst env c0 =  
    let s0 = (fun this -> self#zero) c0 in  
    s0  
  method visit_EAdd env c0 c1 =  
    let s0 = self#visit_expr env c0 in  
    let s1 = self#visit_expr env c1 in  
    self#plus s0 s1  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd (c0, c1) ->  
      self#visit_EAdd env c0 c1  
end
```



look Ma,  
full type inference!



class is polymorphic  
in env and monoid

## Using a “reduce” visitor

**Inherit** the visitor, **inherit** a monoid, **override** one or more methods:

```
let size : expr -> int =
  let v = object
    inherit [_] reduce as super
    inherit [_] VisitorsRuntime.addition_monoid
    method! visit_expr env e =
      1 + super # visit_expr env e
  end in
  v # visit_expr ()
```

This size computation remains **unchanged** if more expression forms are added.



## What we have seen so far

- ▶ **Several built-in varieties**: iter, map, endo, reduce, mapreduce, fold
- ▶ **Arity two**, too: iter2, map2, reduce2, mapreduce2, fold2
- ▶ **Monomorphic** visitor methods, **polymorphic** visitor classes
- ▶ All types **inferred**

## Support for parameterized data types

We wish to traverse parameterized data types, too.

- ▶ But: how does one traverse a subtree of type 'a?

Two approaches are supported:

- ▶ declare a **virtual visitor method** `visit_'a`
  - ▶ 'a is treated as a fixed/unknown type, not really as a parameter
- ▶ pass a **function** `visit_'a` to every visitor method.
  - ▶ allows / requires methods to be polymorphic in 'a
  - ▶ more compositional

In this talk: monomorphic generated methods, polymorphic hand-written methods.

## A visitor for a parameterized type

Here is a “monomorphic-method” visitor for a parameterized type:

```
type 'info expr_node =  
  | EConst of int  
  | EAdd of 'info expr * 'info expr  
and 'info expr =  
  { info: 'info; node: 'info expr_node }  
[@@deriving visitors { variety = "map" }]
```

```
class virtual ['self] map = object (self : 'self)  
  inherit [_] VisitorsRuntime.map  
  method virtual visit_'info : _  
  method visit_EConst = ...  
  method visit_EAdd = ...  
  method visit_expr_node = ...  
  method visit_expr env this =  
    let r0 = self#visit_'info env this.info in  
    let r1 = self#visit_expr_node env this.node in  
    { info = r0; node = r1 }  
end
```

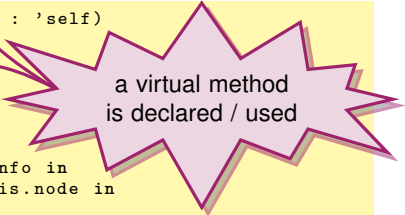
The type of `visit_'info` is `'env -> 'info1 -> 'info2`.

## A visitor for a parameterized type

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  | EConst of int  
  | EAdd of 'info expr * 'info expr  
and 'info expr =  
  { info: 'info; node: 'info expr_node }  
[@@deriving visitors { variety = "map" }]
```

```
class virtual ['self] map = object (self : 'self)  
  inherit [_] VisitorsRuntime.map  
  method virtual visit_'info : _  
  method visit_EConst = ...  
  method visit_EAdd = ...  
  method visit_expr_node = ...  
  method visit_expr env this =  
    let r0 = self#visit_'info env this.info in  
    let r1 = self#visit_expr_node env this.node in  
    { info = r0; node = r1 }  
end
```



a virtual method  
is declared / used

The type of `visit_'info` is `'env -> 'info1 -> 'info2`.

## Using a visitor for a parameterized type

This visitor can map **undecorated** expressions to **decorated** expressions:

```
let number (e : _ expr) : int expr =
  let v = object
    inherit [_] map
    val mutable count = 0
    method visit_'info _env _info =
      let c = count in count <- c + 1; c
    end in
  v # visit_expr () e
```

and vice-versa:

```
let strip (e : _ expr) : unit expr =
  let v = object
    inherit [_] map
    method visit_'info _env _info = ()
  end in
  v # visit_expr () e
```

The visitor class is **polymorphic** in 'env, 'info1 and 'info2.

## A “mapreduce” visitor for a parameterized type

Here is another variety of visitor for this parameterized type:

```
type 'info expr_node =  
  | EConst of int  
  | EAdd of 'info expr * 'info expr  
and 'info expr =  
  { info: 'info; node: 'info expr_node }  
[@@deriving visitors { variety = "mapreduce" }]
```

a “mapreduce” visitor  
is requested

```
class virtual ['self] mapreduce = object (self : 'self)  
  inherit [_] VisitorsRuntime.mapreduce  
  method virtual visit_'info : _  
  method visit_EConst = ...  
  method visit_EAdd env c0 c1 =  
    let r0, s0 = self#visit_expr env c0 in  
    let r1, s1 = self#visit_expr env c1 in  
    EAdd (r0, r1), self#plus s0 s1  
  method visit_expr_node = ...  
  method visit_expr = ...  
end
```

Every method returns a **pair** of a subtree and a summary.

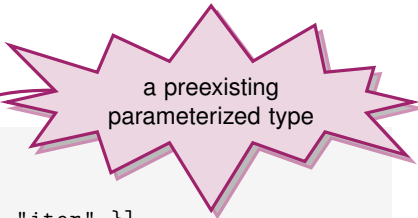
## Using a visitor for a parameterized type

This visitor can **annotate** every subexpression **with its size**:

```
let annotate (e : _ expr) : int expr =
  let v = object
    inherit [_] mapreduce as super
    inherit [_] VisitorsRuntime.addition_monoid
    method! visit_expr env { info = _; node } =
      let node, size = super#visit_expr_node env node in
      let size = size + 1 in
      { info = size; node }, size
    method visit_'info _env _info =
      assert false (* never called *)
  end in
  let e, _ = v # visit_expr () e in
  e
```

## Visiting preexisting types

Lists can be visited, too.



a preexisting  
parameterized type

```
type expr =  
  | EConst of int  
  | EAdd of expr list  
  [@@deriving visitors { variety = "iter" }]
```

```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    ()  
  method visit_EAdd env c0 =  
    let r0 = self#visit_list self#visit_expr env c0 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd c0 ->  
      self#visit_EAdd env c0  
end
```

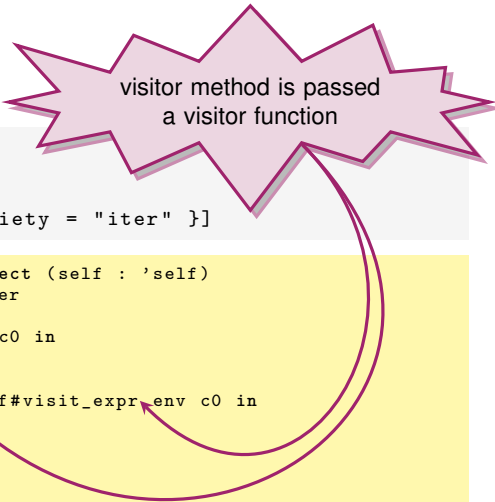


## Visiting preexisting types

Lists can be visited, too.

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type expr =  
  | EConst of int  
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  [@@deriving visitors { variety = "iter" }]
```

```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    ()  
  method visit_EAdd env c0 =  
    let r0 = self#visit_list self#visit_expr env c0 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd c0 ->  
      self#visit_EAdd env c0  
end
```



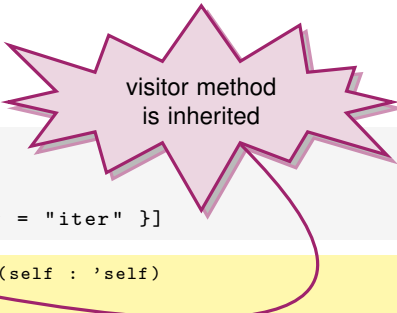
visitor method is passed  
a visitor function

## Visiting preexisting types

Lists can be visited, too.

```
type expr =  
  | EConst of int  
  | EAdd of expr list  
  [@@deriving visitors { variety = "iter" }]
```

```
class virtual ['self] iter = object (self : 'self)  
  inherit [_] VisitorsRuntime.iter  
  method visit_EConst env c0 =  
    let r0 = self#visit_int env c0 in  
    ()  
  method visit_EAdd env c0 =  
    let r0 = self#visit_list self#visit_expr env c0 in  
    ()  
  method visit_expr env this =  
    match this with  
    | EConst c0 ->  
      self#visit_EConst env c0  
    | EAdd c0 ->  
      self#visit_EAdd env c0  
end
```



visitor method  
is inherited

## Predefined visitor methods

The class `VisitorsRuntime.map` offers this method:

```
class ['self] map = object (self)
  (* One of many predefined methods: *)
  method private visit_list: 'env 'a 'b .
    ('env -> 'a -> 'b) -> 'env -> 'a list -> 'b list
  = fun f env xs ->
    match xs with
    | [] ->
      []
    | x :: xs ->
      let x = f env x in
      x :: self # visit_list f env xs
end
```

This method is **polymorphic**, so multiple instances of `list` are not a problem.

## Visitors – a summary



Although they follow fixed patterns, visitors are quite **versatile**.

They are like higher-order functions, only more **customizable** and **composable**.

More fun with visitors:

- ▶ visitors for **open data types** and their fixed points ([link](#));
- ▶ visitors for **hash-consed data structures** ([link](#));
- ▶ **iterators** out of visitors ([link](#)).

In the remainder of this talk:

- ▶ Can we traverse **abstract syntax with binding**?

# Visitors Unchained



## Dealing with binding

Can a visitor traverse **abstract syntax with binding** constructs?

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Can this be done in a **modular** way?

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Exactly which **separation of concerns** should one enforce?



## Dealing with binding

Can a visitor traverse **abstract syntax with binding** constructs?

Can this be done in a **modular** way?

Exactly which **separation of concerns** should one enforce?

- ▶ There are many **binding constructs**,
  - ▶ there are even **combinator languages** for describing binding structure!
- ▶ and many common **operations** on terms,
  - ▶ often specific of one **representation** of names and binders,
  - ▶ sometimes specific of **two** such representations, e.g., **conversions**.
- ▶ Can we insulate the **end user** from this complexity?

We suggest distinguishing **three** principals...



end  
user



A black and white photograph of three men in Western-style clothing. The man in the center is wearing a cowboy hat, sunglasses, and a scarf. The man on the left is wearing a suit and tie. The man on the right is wearing a bowler hat and holding a revolver. The background is white with a red splatter effect. Two orange starburst shapes with red outlines are overlaid on the image, containing the text 'end user' and 'operations specialist'.

end  
user

operations  
specialist

A black and white photograph of three men in Western-style clothing, including hats and scarves. The man in the center is wearing a cowboy hat and sunglasses. The man on the right is holding a rifle. The background is white with red splatters. Three orange starburst shapes with red outlines are overlaid on the image, each containing text.

end  
user

binder  
maestro

operations  
specialist

**The end user**



# Desiderata

The end user wishes:

- ▶ to describe the structure of ASTs in a concise and **declarative** style,
- ▶ not to be bothered with implementation details,
- ▶ possibly to have access to **several representations** of names,
- ▶ to get access to a toolkit of **ready-made operations** on terms.

## Example: abstract syntax of the $\lambda$ -calculus

Let the type `( 'bn, 'term) abs` be a synonym for `'bn * 'term`.

The end user defines his syntax as follows:

```
type ('bn, 'fn) term =
  | TVar of 'fn
  | TLambda of ('bn, ('bn, 'fn) term) abs
  | TApp of ('bn, 'fn) term * ('bn, 'fn) term
[@@deriving visitors { variety = "map";
                      ancestors = ["BindingForms.map"] }]

type raw_term      = (string, string) term
type nominal_term = (Atom.t, Atom.t) term
type debruijn_term = (unit,      int) term
```

He gets **multiple representations** of names.

- ▶ At least two are used in any single application. (Parsing. Printing.)

He gets **visitors** for free. The method `visit_abs` is used at abstractions.


- ▶ `iter`, `map`, `iter2` needed in practice. Focusing on `map` in this talk.



## Example: abstract syntax of the $\lambda$ -calculus

Let the type `( 'bn, 'term) abs` be a synonym for `'bn * 'term`.

The end user defines his syntax as follows:



**provided by the binder maestro**

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## The binder maestro



## An easy job?

Implementing `visit_abs` is the task of our sophisticated binder maestro.

The key is to **extend the environment** when entering the scope of a binder.

Easy?

## An easy job?

Implementing `visit_abs` is the task of our sophisticated binder maestro.

The key is to **extend the environment** when entering the scope of a binder.

Easy? Maybe — yet, the binder maestro:

- ▶ does not know **what operation** is being performed,
- ▶ does not know **what representation(s)** of names are in use,
- ▶ therefore does not know the types of names and environments,
- ▶ let alone **how** to extend the environment.

What he knows is **where** and **with what names** to extend the environment.

## A convention

The binder maestro agrees on a **deal** with the operations specialist.

*“I tell you when to extend the environment; you do the dirty work.”*

The binder maestro **calls** a method which the operations specialist **provides**:

```
(* A hook that defines how to extend the environment. *)  
method private virtual extend: 'env -> 'bn1 -> 'env * 'bn2
```

This is a bare-bones **API** for describing binding constructs.

## Visiting an abstraction

The class `BindingForms.map` offers the method `visit_abs`:

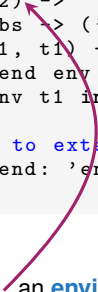
```
class virtual ['self] map = object (self : 'self)
  (* A visitor method for the type abs. *)
  method private visit_abs: 'term1 'term2 . _ ->
    ('env -> 'term1 -> 'term2) ->
    'env -> ('bn1, 'term1) abs -> ('bn2, 'term2) abs
  = fun _ visit_'term env (x1, t1) ->
    let env, x2 = self#extend env x1 in
    let t2 = visit_'term env t1 in
    x2, t2
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This method:

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## Visiting an abstraction

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    x2, t2
  (* A hook that defines how to extend the environment. *)
  method private virtual extend: 'env -> 'bn1 -> 'env * 'bn2
end
```

This method:

- ▶ takes a **visitor function** for terms, an **environment**,
- ▶ an abstraction, i.e., a **pair** of a name and a term, and
- ▶ returns a pair of a **transformed name** and a **transformed term**.

## Visiting an abstraction

That's **all there is** to single-name abstractions.

More binding constructs later on...

For now, let's turn to the final participant.

## The operations specialist



## A toolbox of operations

There are many operations on terms that the end user might wish for:

- ▶ testing terms for **equality** up to  $\alpha$ -equivalence,
- ▶ finding out which names are **free** or **bound** in a term,
- ▶ applying a **renaming** or a **substitution** to a term,
- ▶ **converting** a term from one representation to another,
- ▶ (plus application-specific operations.)

## Implementing an operation

To implement one operation, the specialist decides:

- ▶ the **types** of names and environments,
- ▶ **what to do** at a **free name** occurrence,
- ▶ **how to extend** the environment when entering the scope of a **bound name**.

## Implementing `import`

As an example, let's implement `import`, which converts raw terms to nominal terms.

1. An import environment maps strings to atoms:

```
module StringMap = Map.Make(String)
type env = Atom.t StringMap.t
let empty : env = StringMap.empty
```

## Implementing `import`

2. When the scope of  $x$  is entered, the environment is extended with a mapping of the string  $x$  to a fresh atom  $a$ .

```
let extend (env : env) (x : string) : env * Atom.t =  
  let a = Atom.fresh x in  
  let env = StringMap.add x a env in  
  env, a
```

(An **atom** carries a unique integer identity.)

This is true regardless of which binding constructs are traversed.



## Implementing `import`

3. When an occurrence of the string `x` is found, the environment is looked up so as to find the corresponding atom.

```
exception Unbound of string
let lookup (env : env) (x : string) : Atom.t =
  try StringMap.find x env
  with Not_found -> raise (Unbound x)
```

## Implementing `import`

The previous instructions are grouped in a little class — a “kit”:

```
class ['self] map = object (_ : 'self)
  method private extend      = extend
  method private visit_'fn = lookup
end
```

This is `KitImport.map`.

That's all there is to it... **but...**

The end user must **work**  
a little bit to **glue** everything together...



The end user must **work**  
a little bit to **glue** everything together...

... and may feel **slightly annoyed**.



## Typical glue

For one operation, the end user must write 5 lines of glue code:

```
let import_term env t =  
  (object  
    inherit [_] map          (* generated by visitors *)  
    inherit [_] KitImport.map (* provided by AlphaLib *)  
  end) # visit_term env t
```

For 15 operations, this hurts.

**Functors** can help in simple cases, but are not flexible enough.

**Macros** help, but are ugly. Is there a better way?

## Towards advanced binding constructs



## Defining new binding constructs

There are **many binding constructs** out there.

- ▶ “let”, “let rec”, patterns, telescopes, ...

We have seen how to **programmatically** define a binding construct.

Can it be done in a more **declarative** manner?

## A domain-specific language

Here is a little language of **binding combinators**:

$t$	$::=$	...	sums, products, free occurrences of names, etc.
		$\text{abstraction}(p)$	a pattern, with embedded subterms
$p$	$::=$	...	sums, products, etc.
		$\text{binder}(x)$	a binding occurrence of a name
		$\text{outer}(t)$	an embedded term
		$\text{rebind}(p)$	a pattern in the scope of any bound names on the left

Inspired by  $C\alpha\text{ml}$  (F.P., 2005) and Unbound (Weirich et al., 2011).



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		$\text{inner}(t)$	— sugar for $\text{rebind}(\text{outer}(t))$

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## A domain-specific language

Here is a little language of **binding combinators**:

$t$	::=	...	sums, products, free occurrences of names, etc.
		abstraction( $p$ )	a pattern, with embedded subterms
		bind( $p, t$ )	— sugar for abstraction( $p \times \text{inner}(t)$ )
$p$	::=	...	sums, products, etc.
		binder( $x$ )	a binding occurrence of a name
		outer( $t$ )	an embedded term
		rebind( $p$ )	a pattern in the scope of any bound names on the left
		inner( $t$ )	— sugar for rebind(outer( $t$ ))

Inspired by  $C\alpha$ ml (F.P., 2005) and Unbound (Weirich et al., 2011).

## Implementation

These primitive constructs are just annotations:

```
type 'p abstraction = 'p
type 'bn binder = 'bn
type 't outer = 't
type 'p rebind = 'p
```

Their presence triggers calls to appropriate (hand-written) `visit_` methods.

## Implementation

While visiting a pattern, we keep track of:

- ▶ the **outer environment**, which existed outside this pattern;
- ▶ the **current environment**, extended with the bound names encountered so far.

Thus, while visiting a pattern, we use a richer type of **contexts**:

```
type 'env context = { outer: 'env; current: 'env ref }
```

— Not every visitor method need have the same type of environments!

With this in mind, the implementation of the `visit_` methods is straightforward...

# Implementation

This code takes place in a map visitor:

```
class virtual ['self] map = object (self : 'self)
  method private virtual extend: 'env -> 'bn1 -> 'env * 'bn2
    (* The four visitor methods are inserted here... *)
end
```

1. At the root of an abstraction, **a fresh context** is allocated:

```
method private visit_abstraction: 'env 'p1 'p2 .
  ('env context -> 'p1 -> 'p2) ->
  'env -> 'p1 abstraction -> 'p2 abstraction
= fun visit_p env p1 ->
  visit_p { outer = env; current = ref env } p1
```

# Implementation

2. When a bound name is met, the **current** environment is **extended**:

```
method private visit_binder: _ ->
  'env context -> 'bn1 binder -> 'bn2 binder
= fun visit_'bn ctx x1 ->
  let env = !(ctx.current) in
  let env, x2 = self#extend env x1 in
  ctx.current := env;
  x2
```

## Implementation

3. When a term that is **not in the scope** of the abstraction is found, it is visited in the **outer** environment.

```
method private visit_outer: 'env 't1 't2 .
  ('env -> 't1 -> 't2) ->
  'env context -> 't1 outer -> 't2 outer
= fun visit_t ctx t1 ->
  visit_t ctx.outer t1
```

## Implementation

4. When a subpattern marked `rebind` is found, the **current** environment is installed as the **outer** environment.

```
method private visit_rebind: 'env 'p1 'p2 .
  ('env context -> 'p1 -> 'p2) ->
  'env context -> 'p1 rebind -> 'p2 rebind
= fun visit_p ctx p1 ->
  visit_p { ctx with outer = !(ctx.current) } p1
```

This affects the meaning of `outer` inside `rebind`.



## Example use: telescopes

A dependently-typed  $\lambda$ -calculus whose  $\Pi$  and  $\lambda$  forms involve a telescope:

```
#define tele ('bn, 'fn) tele
#define term ('bn, 'fn) term
type tele =
  | TeleNil
  | TeleCons of 'bn binder * term outer * tele rebound
and term =
  | TVar of 'fn
  | TPi of (tele, term) bind
  | TLam of (tele, term) bind
  | TApp of term * term list
[@@deriving visitors {
  variety = "map";
  ancestors = ["BindingCombinators.map"]
}]
```

# Conclusion



## Conclusion

Visitors are **powerful**.

Visitor classes are **partial**, **composable** descriptions of operations.

Visitors **can** traverse **abstract syntax with binding**.

- ▶ Syntax, binding forms, operations can be **separately** described.
- ▶ Syntax and even binding forms can be described in a **declarative** style.
- ▶ Open-ended, customizable approach.

Limitations:

- ▶ Macros are ugly.
- ▶ No proofs.
- ▶ Some operations may not fit the visitor framework;
- ▶ Some binding forms do not easily fit in the low-level framework or in the higher-level DSL, e.g., Unbound's *Rec*.